



Three Essays in Macroeconomics and International Finance

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Three Essays in Macroeconomics and International Finance

A dissertation presented
by
Vania Atanassova Stavrakeva
to
The Department of Economics
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of
Economics

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Abstract

This dissertation includes three chapters. The first chapter studies the question of whether countries with different fiscal capacity should optimally have different ex-ante minimum bank capital requirements. In an environment with endogenously incomplete markets and overinvestment because of moral hazard and pecuniary externalities, I show that countries with larger fiscal capacity should have lower minimum ex-ante bank capital requirements. I also show that, in addition to the minimum capital requirement, regulators in countries with a concentrated financial sector and large fiscal capacity (which are also countries with strong moral hazard) should impose a limit on the amount of liquidity pledged by financial institutions in a crisis state (for example, restrict the amount of put options/CDS contracts sold by financial institutions). The second chapter studies the welfare implications of a concentrated, imperfectly competitive banking sector, which faces a bank net worth constraint in a small open economy (SOE) environment. There are two standard sources of inefficiency — pecuniary externalities, which lead to overinvestment, and a standard monopolistic underinvestment force. I show that the optimal policy instruments include subsidies on firm borrowing costs in certain periods and capital account controls in others, which is a good proxy for the behavior of emerging markets. For every country, there exists a financial sector with a particular banking sector concentration, for which the inefficiencies offset each other and no government intervention is required in some periods. Furthermore, this paper documents a novel theoretical result — the interaction between future binding bank net worth constraints and dynamic (future) underinvestment could lead to ex-ante overinvestment even in economies with a single monopolistic bank where there are no pecuniary externalities. The last third chapter, which is coauthored with Kenneth Rogoff, evaluates a new class of exchange rate forecasting studies, which claim that structural models are getting closer to being able to forecast exchange rates at short horizons. We argue that misinterpretation of some new out-of-sample tests for nested models, over-reliance on asymptotic test statistics, and failure to sufficiently check robustness to alternative time windows have led many studies to overstate even the relatively thin positive results that have been found.

Contents

1	Optimal Bank Regulation and Fiscal Capacity	1
1.1	Introduction	1
1.2	Model Set-Up	7
1.2.1	Bankers	7
1.2.2	Consumers	9
1.2.3	Policy Maker	10
1.2.4	Assumptions	12
1.3	Solving the Model	13
1.3.1	The Problem of Banker i	13
1.3.2	Constrained Central Planner's Problem Without Commitment	21
1.4	Overinvestment	24
1.5	Decentralize the Constrained Central Planner's Allocation	27
1.6	Comparative Statics of Optimal Regulation With Respect to Fiscal Capacity	32
1.7	A "Price" Instrument Versus A "Quantity" Instrument	35
1.8	Further Discussion and Conclusion	37
2	Welfare Implications of the Structure of the Banking Sector in a Small Open Economy	39
2.1	Introduction	39
2.2	Model	47
2.2.1	The Problem of the Representative Entrepreneur	48
2.2.2	Banker i 's Optimization Problem	50
2.2.3	Constrained Central Planner's Problem	56

2.3	Central Planner's Allocation vs Decentralized Equilibrium	58
2.3.1	Bank Net Worth Constraint Never Binds	59
2.3.2	Binding Bank Net Worth Constraint In a Crisis	60
2.4	Decentralize the Constrained Central Planner's Allocation	63
2.5	Further Discussion	68
3	The Continuing Puzzle of Exchange Rate Forecasting (Joint with Kenneth Rogoff)	70
3.1	Introduction	70
3.2	Definition of a "Good" Exchange Rate Forecast	73
3.3	Minimum Mean-Square Forecast Error Tests: Theil's U (TU), Diebold–Mariano/West (DMW), Clark – West (CW), Clark – McCracken (ENC-NEW)	74
3.3.1	CW/ENC-NEW – Not Always Minimum MSFE Tests	75
3.3.2	The Asymptotics of CW Are Well Defined Only in the Rolling Case	75
3.3.3	Bootstrapped Tests Are Relatively Better Sized and More Powerful	76
3.4	Robustness With Respect to Alternative Test Statistics	76
3.4.1	Engel, Mark and West (2007) — The Monetary Model	77
3.4.2	Molodtsova and Papell (2009) — Heterogeneous Symmetric Taylor Rule with Smoothing	80
3.4.3	Gourinchas and Rey (2007) — External Balance Model	82
3.5	Robustness with Respect to Different Forecast Windows	84
3.5.1	Engel, Mark and West (2007)	84
3.5.2	Molodtsova and Papell (2009)	85
3.5.3	Gourinchas and Rey (2007)	86
3.6	Can One Do Better? The Importance of Common Cross-Country Shocks	88
3.7	Forecast Pooling – Empirical Example	90
3.7.1	Endogenous vs Exogenous Weights	90
3.7.2	Results After Pooling	91
3.7.3	Robustness of the Pooled Forecast with Respect to Different Forecast Windows - Is Pooling Enough?	93
3.8	Conclusion	94
	Bibliography	97
	A Supplement to Chapter 1	106
A.1	Appendix	106

A.1.1	The Problem of the Consumer	107
A.1.2	N Banks – No Commitment	108
A.1.3	Constrained Central Planner’s Problem – No Commitment	112
A.1.4	Proofs	114
B	Supplement to Chapter 2	141
B.1	Appendix	141
B.1.1	The Problem of the Entrepreneur	141
B.1.2	The Problem of Banker i	142
B.1.3	Constrained Central Planner’s Problem	145
B.1.4	Ramsey Problem	147
B.1.5	Proofs	151
C	Supplement to Chapter 3	156
C.1	Appendix	156
C.1.1	Minimum MSFE Out-of-Sample Test Statistics	156
C.1.2	Proofs: The New Out-of-Sample Tests for Nested Models	157
C.1.3	Bootstrap	162

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Chapter 1

Optimal Bank Regulation and Fiscal Capacity

1.1 Introduction

This paper addresses the question of whether countries with different abilities to bail-out their banking system during a financial crisis should have more or less stringent ex-ante minimum bank capital requirements.

Countries vary widely in their ability to bail out their financial system. A bank bail-out can be financed either by taxing, borrowing or printing money if independent monetary policy is available. In equilibrium, a country will use all of those instruments up to the point where the marginal cost of each one of them is equalized and is, in turn, equal to the marginal cost of the bail-out. All else constant, the marginal cost of the bail-out will be larger, if a country has a smaller tax base, higher cost of sovereign borrowing during a crisis or does not have an access to an independent monetary policy. I refer to the marginal cost of an extra dollar of bail-out as the fiscal capacity of a country. For example, as shown in Figure 1.1, a country like Switzerland, which has a large banking sector relative to the GDP of the country is relatively more fiscally constrained than the United States and would optimally choose to bail out a smaller fraction of its banking sector during a crisis.¹

¹Using BankScope data, in 2007, in the US, the assets of the banking sector were 120% of the GDP of the country. This number also includes foreign subsidiaries/branches in the US such as UBS but does not include subsidiaries/branches of US banks abroad. Even counting the shadow banking sector in the US, the total assets of the banking sector (as defined above) plus the shadow banking system becomes 300% of the US GDP in 2007 (see the Financial Stability Board estimates, 2007). In 2007, in Switzerland, the assets of just the Swiss branches of UBS and Credit Swiss were 700% of the GDP of the country.

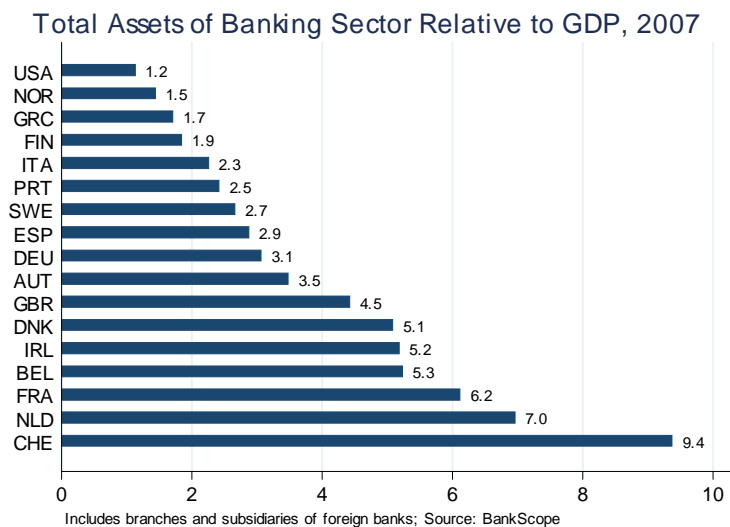


Figure 1.1

During the 2011-2012 European sovereign debt crisis, Greece and Spain are two prime examples of countries that were fiscally constrained since they faced a high cost of sovereign borrowing and were limited in the amount of bail-outs they could provide to their financial sectors.²

Despite clear differences in the ability of countries to provide bank bail-outs during financial crises, most countries have synchronized their ex-ante minimum bank capital requirements following the Basel Accords. The minimum bank capital requirement constrains banks to finance at least a fraction of risky bank assets using equity. This fraction is referred to as the minimum bank capital ratio.³ In response to the 2007-2008 financial crisis, in 2011, Swiss regulators deviated from the Basel I norm of 8 percent minimum bank capital ratio by significantly increasing the minimum bank capital requirement to 19 percent. This triggered a vigorous debate as to whether the actions of Swiss regulators were optimal.

This paper provides a theoretical justification for heterogeneous cross country minimum bank capital requirements, a hypothesis that contrasts with the current synchronization of minimum bank capital requirements across most countries. More specifically, I show that in a model with both pecuniary externalities and moral hazard, countries with larger fiscal capacity should have lower ex-ante minimum bank capital requirements. Furthermore, a second interesting and novel result emerges, which is that countries with large fiscal capacity and a concentrated banking sector should also impose a limit on bank liabilities in a crisis,

²In 2011, the 10 year sovereign debt interest rate was 5.4% in Spain and 35% in Greece.

³The World Bank survey on bank regulation indicates that, in 2010, the majority of countries had a minimum bank capital requirement of eight percent, which is the capital ratio recommended by Basel I. Among the high income OECD countries, 25 out of 27 had a minimum bank capital requirement of 8 percent (the exceptions are Israel and Estonia).

when a government bail-out is anticipated. In particular, it would be optimal to limit the payments bankers promise in a crisis state by selling put option contracts such as CDS contracts, for example.

I build a three period model in the spirit of Lorenzoni (2008). Bankers are modelled as entrepreneurs that have access to a linear production technology. They borrow every period using short term state contingent contracts subject to borrowing constraints and invest in every period. The project has to be refinanced in the middle period in order to remain productive. If a crisis state occurs, in order to refinance the project, bankers are forced to sell part of the capital to the less productive outside sector, which generates fire sales. The government can intervene during a crisis and provide a bail-out to the bankers by taxing the consumers. However, taxing is costly, and the cost depends on the fiscal capacity of the country. The combination of future fire sales and the assumption that the banking sector is more productive than consumers generates pecuniary externalities in the spirit of Lorenzoni (2008), which lead to overinvestment. Bankers do not internalize the fact that the more they invest ex-ante, the larger the fire sale of financial assets during a future crisis is, which tightens the budget constraints of the other bankers and increases the inefficient transfer of resources from the bankers — the more productive agents — to the consumers — the less productive agents. The presence of ex-post anticipated bail-outs leads to moral hazard if the financial sector has a finite number of banks and is a second source of overinvestment. When banks expect a bail-out in the future in the event of a crisis, if they are not infinitesimally small, they overinvest because they internalize the fact that larger ex-ante investment increases the size of the fire sale and, therefore, increases the bail-out received. The moral hazard is stronger, the more concentrated the banking sector is and the larger the fiscal capacity of the country is.

The key result of the paper — that countries with larger fiscal capacity should have lower ex-ante minimum bank capital requirements — is driven by the assumptions which are at the core of the pecuniary externalities and the overinvestment — the presence of fire sales combined with the assumption that the banking sector is more productive than the outside sector. The intuition behind the result is the following. For a given amount of ex-ante investment, the countries that can provide a smaller bail-out during a crisis will have a larger fire sale, and the transfer of capital from the more productive sector to the less productive sector will be larger. Therefore, the Central Planner in more fiscally constrained countries perceives ex-ante investment as less attractive, and he optimally chooses to invest less in period zero than does the Central Planner of a country with a larger fiscal capacity. Since the period zero investment chosen by the Central Planner and the optimal minimum bank capital ratio are inversely related, smaller fiscal capacity implies optimally higher ex-ante minimum bank capital ratio. In summary, countries with larger fiscal capacity can afford to prop up prices by more in a crisis and alleviate any inefficiencies arising from fire sales. As a result,

they can "afford" to have larger investment booms ex-ante.

Since a larger bail-out will lead to a stronger moral hazard if the banks are not infinitesimally small, intuitively, one would expect that larger fiscal capacity would imply higher (not lower) minimum bank capital requirements. The reason why this intuition proves to be incorrect relies crucially on the fact that the ex-ante regulatory instrument considered, a minimum bank capital requirement, is a "quantity" policy instrument and on the assumption that the policy maker has a sufficient number of instruments to replicate the constrained Central Planner's allocation. By setting the minimum bank capital ratio, the policy maker can directly determine the amount of bank investment. The moral hazard does not play a role since it does not affect the optimal amount of investment chosen by the Central Planner. The moral hazard would enter only the first order conditions of the banker from the decentralized equilibrium since the banker internalizes the benefit but not the cost of the bail-out. However, if the constrained Central Planner's allocation is replicated, when the regulatory instrument is a "quantity" instrument, the first order condition of the banker from the decentralized problem is no longer relevant. This will differ if a "price" instrument was used instead, such as a tax on ex-ante investment.⁴

The second key result of this paper is that countries with strong moral hazard, which are the countries with a few banks and a concentrated banking sector, should impose a second ex-ante regulatory instrument, which limits the liabilities of bankers in a crisis state when a bail-out is expected. The moral hazard in this model works through two different channels. Bankers are tempted to overinvest ex-ante, and if the moral hazard is strong enough, they are also tempted to pledge too high of a payment in the crisis state using the state contingent contract, relative to what the Central Planner would optimally do. The intuition is that a larger promised payment in the crisis state leads to a more severe fire sale and, therefore, to a larger expected bail-out. More than one hundred countries worldwide currently have some form of a minimum liquidity requirement and there is little understanding as to why such an instrument will be required, in addition to the minimum bank capital requirement⁵. According to the results of this model, the liquidity that policy makers should be particularly concerned about is the liquidity of the financial sector in a crisis. Therefore, regulating derivative contracts for countries with strong moral hazard would be important.

Related Literature

⁴"Price" instruments, such as a tax on period zero capital, have been considered in the theoretical literature (see Stein (2012), Bianchi (2011) and Jeanne and Korinek (2012)).

⁵The World Bank survey on bank regulation shows that in 2010, 103 out of 127 countries had some form of a minimum ratio on liquid assets, such as a regulatory minimum ratio on liquid assets as a percentage of total balance sheet or deposit base.

There are three key assumptions that lead to the main result of this paper and they are fairly standard in the literature. The first assumption, that bankers face a borrowing constraint, is in the spirit of Kiyotaki and Moore (1997) and is based on the seminal paper by Hart and Moore (1994).⁶ The second assumption is that the banking sector is more productive than the outside sector (the consumers), and it has been used in many papers such as Kiyotaki and Moore (1997), Lorenzoni (2008), Brunnermeier and Sannikov (2013). It can be further justified by the fact that financial institutions are considered more efficient than savers at providing credit to firms, because of their ability to monitor the borrower at a lower cost (for example see Holmstrom and Tirole (1997)). Hence, the value of loans (capital) is higher in the hands of the bankers than in the hands of the consumers. On the empirical side, the importance of the banking sector and the value lost by terminating the relationship between banks and firms are studied by the literature on relationship banking (see Slovin, Sushka, and Polonchek (1993), Peek and Rosengren (1997), Gan (2007), Boot (2000) and Freixas and Rochet (2008) for a literature review). The third key assumption — that the government can circumvent the banker’s borrowing constraint via its power to tax — was used, besides others, by Holmstrom and Tirole (1998) and Gorton and Huang (2004). Similarly to this paper, they also show that such government intervention can be welfare improving.⁷

Starting with the seminal work of Bagehot (1873), moral hazard has been proposed as one of the main reasons for bank regulation. However, a growing literature on financial sector regulation has emerged, which emphasizes the role of fire sales and pecuniary externalities —

Geanakoplos and Polemarchakis (1986), Lorenzoni (2008), Stein (2012), Jeanne and Korinek (2012), He and Kondor (2012). The importance of fire sales during financial crises has been emphasized by many papers starting with the seminal work by Shleifer and Vishny (1992) (see Shleifer and Vishny (2011) for a survey of the literature). I build on the paper by Lorenzoni (2008) who shows how pecuniary externalities can emerge in a microfounded environment^{8,9}. The difference between this paper and the one by Lorenzoni (2008) is that neither he allows for an ex-post bail-out, nor does he ask the question how optimal policy should vary with the fiscal capacity of the country, which is the key contribution of this paper. The only other paper, besides this one, that studies the optimal mix of ex-ante regulation and ex-post bail-out, which has both

⁶Hart and Moore (1994) show that if entrepreneurs can run away with the cash flow and can threaten to withdraw their human capital, they can borrow only against collateral.

⁷Other papers that find ex-post bail-outs to be welfare-improving include Bianchi (2012) and Keister (2012).

⁸There is a large literature on pecuniary externalities where the inefficiency comes from binding borrowing constraints and prices enter the borrowing constraint (for example Stein (2012), Bianchi (2011)). In this paper, as in Lorenzoni (2008), the source of the pecuniary externality will be that bankers do not internalize the fact that their actions are tightening the *budget* constraints of the other bankers, not the *borrowing* constraints.

⁹Lorenzoni (2008) also assumes that in addition to the borrowing constraint on the side of the bankers there is a limited commitment friction on the side of the consumers. I show that as long as the banks are owned by the consumers (an implicit ex-post transfer), limited commitment of the consumers will not be necessary to generate pecuniary externalities.

pecuniary externalities and moral hazard, is the one by Jeanne and Korinek (2012). In contrast to this paper, in Jeanne and Korinek (2012) markets are exogenously incomplete. Endogenous market incompleteness is important to understand the key sources of inefficiency. It is also crucial for some of the key results of the paper such as the result that, for countries with stronger moral hazard, regulators need to control the liabilities of the banking sector in a crisis state. Most importantly, Jeanne and Korinek (2012) do not ask the question raised by this paper: How the optimal mix of ex-ante and ex-post bank regulation should vary with the fiscal capacity of the country?

Some of the recent papers which also find that there is a role for minimum liquidity regulations due to moral hazard are Farhi and Tirole (2012), Acharya, Shin, and Yorulmazer (2011), Repullo (2005) and Keister (2012). In an environment with a non-targeted bail-out in the form of lowering the borrowing rate of banks, Farhi and Tirole (2012) show that there are complementarities in the actions of bankers and, as a result, multiple equilibria. If banks expect low interest rates during crises, they might end up holding too little of the safe asset which, in equilibrium, will force the policy maker to keep interest rates low. As a result, minimum liquidity requirements can improve welfare. In a Diamond and Dybvig (1983) environment with multiple equilibria, Keister (2012) shows that an expected government bail-out ex-post leads to moral hazard and to bankers choosing lower liquidity ex-ante relative to what the Central Planner would choose.

The Basel Accord recommendation of synchronized regulation is often justified by the idea of creating a "level-playing" field for banks. For a summary of the bank regulation literature see Santos (2001). Acharya (2002) studies whether minimum capital requirements should be synchronized across countries given the presence of different bank closure policies and he argues in favor of heterogeneous cross country bank regulation. Bengui (2011) builds a two country model and discusses optimal bank regulation in an environment with pecuniary externalities where he emphasizes the importance of international coordination.

Finally the paper also relates to the literature about different types of regulatory instruments pioneered by Weitzman (1974). According to Weitzman (1974), if the policy maker has an access to state contingent policy instruments, she can replicate the constrained Central Planner's allocation using either a "price" or a "quantity" instrument. However, comparative statics can be very different depending on which type of instruments is used, as I show in this paper.

The structure of the paper is the following. In Section 1.2, I present the set up of the model. Section 1.3 provides the solution of the decentralized equilibrium and the constrained Central Planner's problem. Section 1.4 proves that there is overinvestment in this economy while Section 1.5 shows how the constrained Central Planner's allocation can be decentralized. Section 1.6 proves the key result of the paper — that

countries with larger fiscal capacity should have lower ex-ante minimum bank capital requirements. Section 1.7 compares "quantity" regulatory instruments to "price" regulatory instruments. Section 1.8 concludes and provides further discussion.

1.2 Model Set-Up

There are three agents in the economy — consumers, bankers/entrepreneurs and the government. The banks are owned by the consumers and the consumers receive all the profits in the form of dividends.¹⁰¹¹ Hence, all the dividends are paid out to the consumers who are risk neutral. The discount factor between periods is one. The model is a three period model where $t = 0, 1, 2$ and there is no discounting. There is uncertainty only in the middle period, $t = 1$. In $t = 1$ there are two states of nature — a high state and a low state. The period zero probability of the high state occurring is π_h and the period zero probability of the low state occurring is π_l where $\pi_l + \pi_h = 1$. In equilibrium, the fire sale will occur only in the low state in period one, which is what I will refer to as the crisis state. There are two goods — a capital good and a consumption good where the consumption good is the numeraire good. Each period and state of nature, the consumption good can be transformed into a capital good one-to-one. There is no storage technology either for the capital good or for the consumption good. The capital good has to be employed in a production technology in every period and the consumption good is perishable.

1.2.1 Bankers

Production Technology

Assume that there are N bankers/entrepreneurs. Every banker has an access to a bank specific production technology. In $t = 0$, banker i has to choose the amount he invests given by k_0^i . In $t = 1$ and state s , the project produces $a_{1s}k_0^i$ units of the consumption good where $a_{1l} < a_{1h}$. In $t = 1$, in order for the capital stock, k_0^i , to remain productive, it has to be refinanced. The banker has to invest an additional amount of

¹⁰The model can be easily changed so that some weight is placed on the bankers and they consume as well. In addition, I can impose a requirement for an ex-ante welfare Pareto improvement for both the bankers and the consumers. The results remain unchanged as long as the government has an access to ex-post lump sum transfers from the bankers to the consumers.

¹¹Alternatively, one can think of this set up as there being a representative family that splits into bankers and consumers in the beginning of period zero and the bankers are given the exogenous starting capital n . The bankers and consumers reunite in $t = 2$ and consume jointly. The bankers borrow from consumers that are not members of their family. The latter interpretation is in the spirit of Gertler and Kiyotaki (2010).

$\gamma < 1$ per every unit of period zero capital, γk_0^i in total.¹² Otherwise, the capital depreciates one hundred percent. I assume that the resale price of capital in period one, q_{1s} , is always greater than γ and, hence, all the capital is refinanced. In addition, in $t = 1$, banker i also decides on the period one scale of the project, k_{1s}^i . If the banker has the resources and finds it optimal, he can increase the scale of the project, $k_{1s}^i > k_0^i$. If he does not have the resources or it is not optimal to do so, he can end up investing less than the period zero investment, $k_{1s}^i < k_0^i$. The capital sold by banker i in period one is $k_{1s}^{i,T} = \min\{0, k_0^i - k_{1s}^i\}$. I will refer to $k_{1s}^{i,T}$ as the fire-sold capital and the aggregate amount of fire-sold capital is defined as an average $k_{1s}^T = \sum_{i=1}^N \frac{1}{N} k_{1s}^{i,T}$. Using averages, instead of simple sums, implies that the model can be intuitively mapped to the case with a continuum of banks distributed uniformly on $[0, 1]$, when one takes the limit of $N \rightarrow \infty$.¹³ In equilibrium, there will be a fire sale only in the crisis state. Finally, In $t = 2$, there is no further uncertainty and the amount invested in $t = 1$ produces Ak_{1s}^i , where $A > 0$, and the capital can be sold to the consumers for the price of q_{2s} , after the banker pays the refinancing cost γ .

State Contingent Debt Contracts Subject to a Borrowing Constraint

In $t = 0$, each banker is endowed with n units of the consumption good. n is an exogenous parameter and in this model represents the equity of the banker. Banker i can also borrow from the consumers but credit markets are imperfect due to an agency friction in the spirit of Hart and Moore (1994) and Kiyotaki and Moore (1997). I assume that the state of nature is verifiable and contractible but the banker can always run away with the cash flow in $t = 1$, $a_{1s}k_0$, and in $t = 2$, Ak_{1s} . However the consumer can seize the collateral and resell it. In period zero, banker i can borrow only against the value of period one collateral. The reason is that once in period one, banker i gives an enforceable "take-it-or-leave-it" offer to the consumer — either take $\theta(q_{1s} - \gamma)k_0^i$ in period one as a payment or the bank will be closed (the banker will withdraw his human capital) and no output will be produced in period two. $1 - \theta$ is the fraction of the value of the collateral that has to be paid in legal fees if the consumer has to seize the collateral.¹⁴ If the bank is closed, the consumer will withhold the capital, pay the legal fees and the refinancing cost and resell the capital, which will generate $\theta(q_{1s} - \gamma)k_0^i$ units of the consumption good. In equilibrium, the consumer will always accept the "take-it-or-leave-it" offer. Anticipating that, in $t = 0$, the consumer will write only a short term state contingent debt contract with the banker subject to the collateral constraint. Once the banker repays his old debt in $t = 1$, he can enter a new collateralized contract with the consumers. Since, in period two, he will

¹²One can think of the refinancing cost as a long term project that requires refinancing in order to remain productive (for example workers have to be paid, more equipment has to be purchased etc). Alternatively I could have set γ to zero and assumed that $a_{1t} < 0$ which would be another way to generate a fire sale.

¹³The results do not change if simple sums are used instead of averages.

¹⁴One can set $\theta = 1$ and all the results remain. In the simulations presented in the paper I will set $\theta = 1$.

never pay more than the value of the collateral after legal fees, the maximum amount the banker can borrow in $t = 1$ and state s is $\theta (q_{2s} - \gamma) k_{1s}^i$.

In summary, the contract that emerges in equilibrium is a short-term, state contingent debt contract subject to a collateral constraint on the part of the banker. In $t = 0$, banker i can sell a promise to pay d_{1s}^i units of the consumption good in $t = 1$, state s , at the price $\pi_s p_{1s}$. This implies that the total period zero borrowing of banker i is $\sum_s \pi_s p_{1s} d_{1s}^i$. Also in $t = 1$ state s , banker i can sell a new promise to pay d_{2s}^i units of the consumption good in $t = 2$ state s at the price p_{2s} . As a result, his period one, state s borrowing is $d_{2s}^i p_{2s}$. The prices, p_{1s} and p_{2s} , will be determined in equilibrium. The amount that the banker can borrow is limited by the state contingent value of his collateral; $d_{1s}^i \leq \theta (q_{1s} - \gamma) k_0^i$ and $d_{2s}^i \leq \theta (q_{2s} - \gamma) k_{1s}^i$.

Finally, banker i might receive an additional source of funding in the crisis state in the form of a bail-out. In the low state in period one, banker i receives B_t^i as a transfer from the government, where B_t^i is endogenously determined. Banker i can pay dividends every period and state of nature but he will optimally do so only in the last period, $t = 2$, when he gives all of the profits to the consumers.

1.2.2 Consumers

There is a continuum of risk neutral consumers distributed uniformly over the unit interval. They are the only agents that consume in this economy. In every period and state of nature, every consumer receives an endowment e . He can enter a state contingent contract with each banker both in periods zero and in period one, as described in the previous section. The preferences of the representative consumer are given by

$$U_0^c = c_0 + \sum_s \pi_s (c_{1s} + c_{2s})$$

Consumers also have an access to a production technology which uses capital as an input good and transforms it into the consumption good within the same period. Once the production technology produces the consumption good, the capital depreciates one hundred percent. When modelling the production technology of the consumers in $t = 1$, in order to generate a downward sloping demand for capital, I use an approach similar to many papers in the literature on financial frictions that follow the seminal paper of Kiyotaki and Moore (1997). In $t = 1$, the production technology of the consumers is given by $F(k_{1s}^T)$ where k_{1s}^T is the amount of capital employed, which, in equilibrium, will be the aggregate amount of capital fire-sold by the

bankers. In equilibrium, if the production technology of the consumer is employed, the price of capital will be pinned down by the marginal product of capital, $q_{1s} = F'(k_{1s}^T)$. In $t = 2$, I assume that the production technology is such that one unit of capital is transformed into one unit of consumption.¹⁵ In equilibrium, the price of capital in $t = 0$ and $t = 2$ will be pinned down to one, $q_0 = q_{2s} = 1$.¹⁶

The production technology satisfies the following assumptions: $F(k_{1s}^T)$ is at least three times differentiable on $k_{1s}^T \in (0, \infty)$; $F(k_{1s}^T)$ and $F'(k_{1s}^T)$ are continuous on $\in [0, \infty)$; $F(0) = 0, F'(0) = 1, F''(0) = F'''(0) = 0$, $F''(k_{1s}^T) < 0$ on $k_{1s}^T \in (0, \infty)$ and $\lim_{k_{1s}^T \rightarrow \infty} F'(k_{1s}^T) \geq \gamma$. Assuming that $F'(0) = 1$, and $F''(\cdot) < 0$ implies that the production technology of the consumers will not be used unless there is a fire sale.¹⁷ The assumption $F''(\cdot) < 0$ guarantees that the larger the fire sale is, the lower the price of capital will be, which is a proxy for a downward sloping demand for capital. Finally $\lim_{k_{1s}^T \rightarrow \infty} F'(k_{1s}^T) \geq \gamma$ ensures that the scrapping of capital will be never optimal. Notice that the bankers' production technology is more productive than the consumers'. A detailed solution to the consumer's problem is provided in the Appendix, Section A.1.1. Since consumers are risk neutral, in equilibrium, from the Euler equation, the prices of the state contingent debt contracts will be $p_{1s} = p_{2s} = 1$.

1.2.3 Policy Maker

The policy maker optimizes the ex-ante welfare of consumers who are the owners of the banks. He has an access to ex-ante and ex-post policy instruments. The ex-ante policy instruments are a minimum bank capital requirement and a limit on the payment promised in the crisis state by bankers. The minimum bank capital requirement is defined as $\rho^i \leq \frac{n}{k_0^i}$ where ρ^i is the minimum bank capital ratio of bank i . ρ^i is the minimum fraction of period zero investment that has to be financed using equity. I chose to focus throughout most of the paper on the minimum bank capital requirement as the ex-ante policy instrument used to address overinvestment, because this is the instrument currently implemented by almost all countries worldwide. I allow the policy maker also to explicitly restrict the amount of payments a banker pledges in the crisis state by imposing the constraint $d_{1l}^i \leq \nu^i$. While such an instrument is currently not used in practice, in this

¹⁵The assumption that the production technology is different across periods is for simplification and can be relaxed. However, given that the model is a finite period model, relaxing it would imply that there will be always fire sales in $t = 2$ both in the high and the low state, which will be an unappealing feature of the model. Usually crises are followed by normal times and this would be captured in an infinite period model.

¹⁶The assumption that capital cannot be stored implies that consumers cannot simply purchase capital in $t = 1$ in the low state and keep it until $t = 2$ in order to use their production technology that allows them to transform capital into consumption one-to-one. If that were the case, there would be no fire sales. Also the price of capital is one in period zero since I start the economy with no stock of capital. Therefore, $q_0 = 1$ as long as some investment in capital is made.

¹⁷If there is no fire sale, it will be never profitable for the consumers to transform consumption one-to-one into capital in $t = 1$ in order to use the production technology since the marginal product of capital is less than one.

paper I will show that the presence of such an instrument will allow policy makers of countries with strong moral hazard to improve upon aggregate welfare. In order to map this regulation to contracts used in the real world, policy makers would want to limit the liabilities bankers face in a crisis state due to standard debt contracts as well as derivative contracts such as CDS contracts and the sale of other put options.

The ex-post regulatory instrument is an optimal government bail-out during crises. I implicitly assume that the government can circumvent the collateral constraint of bankers during crises via its power to tax the consumers and transfer resources to the bankers. This is equivalent to allowing the policy maker a bail-out technology during a crisis. For simplicity, I assume that bail-outs are prohibitively costly when there is no crisis due to high political costs of transferring money from taxpayers to the financial system in normal times.¹⁸ The bail-out is financed by taxing the consumers in the crisis state, where the tax levied is T_{1l} . Market clearing implies that $T_{1l} = B_l$. I assume that bailing out the banking system is costly, and this cost can vary across countries. The size of the deadweight loss from the bail-out is given by $\delta(B_l, \chi)$ where $B_l = \sum_{i=1}^N \frac{1}{N} B_l^i$ is the aggregate bail-out¹⁹, B_l^i is the bail-out given to bank i , and $0 \leq \chi < \infty$ is an exogenous parameter that captures how fiscally constrained the country is. *The smaller χ is, the more fiscally constrained the country is (lower fiscal capacity).*

I assume that $\delta(B_l, \chi)$ is a convex and increasing function with respect to the aggregate bail-out, which implies $\frac{\partial \delta(B_l, \chi)}{\partial B_l} > 0$ and $\frac{\partial^2 \delta(B_l, \chi)}{\partial B_l \partial B_l} > 0$. $\frac{\partial \delta(B_l, \chi)}{\partial B_l} > 0$ guarantees that the larger the total size of the bail-out is, the larger the deadweight loss is. $\frac{\partial^2 \delta(B_l, \chi)}{\partial B_l \partial B_l} > 0$ implies that the marginal cost of the bail out increases with the total size of the bail out. The convexity of the deadweight loss is a standard assumption in the public finance literature to capture the cost of distortionary labor taxation in a reduced form way. There are a few additional assumptions. I assume that $\frac{\partial \delta(B_l, \chi)}{\partial \chi} < 0$, which implies that the larger the fiscal capacity of a country is, the smaller the deadweight loss from the bail-out is. Also I assume that $\frac{\partial^2 \delta(B_l, \chi)}{\partial B_l \partial \chi} < 0$ which implies that the marginal cost of taxing is lower, the larger the fiscal capacity is. The final set of assumptions are that $\frac{\partial^3 \delta(B_l, \chi)}{\partial B_l^2 \partial \chi} < 0$ and the deadweight loss of the bail-out, $\delta(B_l, \chi)$, $\frac{\partial \delta(B_l, \chi)}{\partial B_l}$ and $\frac{\partial^2 \delta(B_l, \chi)}{\partial B_l \partial B_l}$ are all equal to zero when $\chi \rightarrow \infty$ and approach infinity when $\chi = 0$. For example, a functional form that satisfies all of these conditions and will be used in the simulations is $\delta(B_l, \chi) = \frac{1}{\chi} B_l^\eta$ where $\eta > 1$.

Notice that instead of introducing the exogenous parameter χ , I could have chosen to model the cost of the bail-out simply as the deadweight loss from taxing scaled by the tax base, e , which would have a

¹⁸One can relax the assumption that there are bail-outs only in the crisis state. The result that larger fiscal capacity implies a lower ex-ante minimum bank capital requirement remains.

¹⁹This assumption captures the fact that it is the aggregate bail-out that affects the marginal cost of government borrowing or taxing.

closer mapping to the public policy literature.²⁰ However, in reality, bail-outs are financed in three different ways — by taxing the residents of the country during the financial crisis, by sovereign borrowing or by printing money if the government has an access to an independent monetary policy. The costs associated with sovereign borrowing, taxing and printing money can be realistically proxied using convex cost functions. In equilibrium, a country which has access to all three instruments will use all of them up to the point where the marginal cost of taxing is equal to the marginal cost of printing money and to the marginal cost of sovereign borrowing, which will, in turn, equal the marginal cost of bail-out. As a result, even if I were to write a full blown model with sovereign borrowing, taxing and printing of money, the marginal cost of the bail-out would be a sufficient statistics. Therefore, I choose to model the marginal cost of the bail-out, $\frac{\partial \delta(B_t, \chi)}{\partial B_t}$, in a reduced form way in order to obtain analytical results. The parameter χ captures the ability of a country to provide an extra dollar of bail-out, which I call fiscal capacity in this paper.

The reason why the government would optimally choose to provide a bail-out to the banking sector, even though the bail-out is costly, is that the bail-out will reduce the size of the fire sale during financial crises. As a result, less capital will be transferred from the more productive sector (the banking sector) to the less productive sector (the consumers). The consumers end up benefiting because they receive the profits of the banks in the last period in the form of dividends. As I show later, the optimal bail-out will be determined in equilibrium by equating the marginal cost of the bail-out to the marginal benefit. The optimal bail-out will be a function of the fiscal capacity of the country, χ . The larger the fiscal capacity is, the smaller the deadweight loss is, and, hence, the optimal bail-out will be larger relative to a country with smaller fiscal capacity. A country with infinite fiscal capacity ($\chi \rightarrow \infty$) will have a zero deadweight loss from the bail-out. A country with no fiscal capacity ($\chi = 0$) will have an infinite deadweight loss from the bail-out. This will be a country that cannot print money, finds it too costly or impossible to tax and it is also completely shut-off from foreign debt markets.

1.2.4 Assumptions

In addition to the assumptions made so far on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, I also assume that the following inequalities are satisfied. The first assumption ensures that period zero investment does not have expected return greater than one in period one if the return in the low state is zero

$$\pi_h (1 + a_{1h} - \gamma) < 1$$

Assumption 1.1

²⁰If I were to use the following functional form for the deadweight loss from taxing $\delta\left(\frac{B_t}{e}\right)$, then one can interpret χ as e .

If Assumption 1.1 is violated, it will always be optimal to lever to the maximum in period zero and invest as much as possible, which will make the problem trivial. The following assumption ensures that if there is no fire sale, the expected return on period zero investment is greater than the cost

$$1 < \sum_s \pi_s [1 - \gamma + a_{1s}] \quad \text{Assumption 1.2}$$

If Assumption 1.2 was violated, period zero investment would be zero. In order to have a fire sale in the model, it has to be the case that the fraction of the capital value that can be pledged, $\theta(1 - \gamma)$, plus the return to period zero capital in the crisis state, a_{1l} , is less than the refinancing cost of capital, γ . I also assume that the refinancing cost is less than one, which is the highest possible price of capital.

$$a_{1l} + \theta(1 - \gamma) < \gamma < 1 \quad \text{Assumption 1.3}$$

To ensure uniqueness, I also assume²¹

$$\begin{aligned} F'(k_{1l}^T) - \theta(1 - \gamma) + F''(k_{1l}^T) k_{1l}^T &> 0 \\ \frac{\partial (F'(k_{1l}^T) - \theta(1 - \gamma) + F''(k_{1l}^T) k_{1l}^T)}{\partial k_{1l}^T} &= F'''(k_{1l}^T) k_{1l}^T + F''(k_{1l}^T) < 0 \end{aligned} \quad \text{Assumption 1.4}$$

Assumption 1.5 guarantees that in the high state there is never a fire sale (i.e. the return per unit of k_0 is higher than the refinancing cost of k_0)

$$a_{1h} > \gamma \quad \text{Assumption 1.5}$$

Assumption 1.6 guarantees that the optimal bail-out and the amount of resources that can be transferred to the crisis state using the state contingent debt contract are not too large so that there will be a fire sale in the crisis state.

$$\frac{(a_{1l} - \gamma)n}{1 - \theta(1 - \gamma)} + (\delta')^{-1} \left(\frac{A - \gamma}{1 - \theta(1 - \gamma)} \right) < 0 \quad \text{Assumption 1.6}$$

Assumption 1.7 ensures that the return to period zero and period one investment (after the refinancing cost is paid) is non-negative

$$A - \gamma > 0, a_{1s} \geq 0 \quad \text{Assumption 1.7}$$

²¹In models with binding borrowing constraints there might be multiple equilibria because both the demand for and the supply of fireold capital is downward sloping. For details, see Lorenzoni (2008).

1.3 Solving the Model

1.3.1 The Problem of Banker i

In this section I solve the optimization problem of banker i , assuming no commitment and that the policy maker provides an optimal ex-post bail-out during a crisis.

Banker i optimizes the dividend payments to the equity owners (consumers). He takes as given the prices of the state contingent borrowing contracts, $p_{1s} = 1$ and $p_{2s} = 1$, which are determined by the first order conditions of the consumer's problem and are not affected by banker i 's choice variables. However, banker i internalizes the fact that his actions affect the price of capital in the middle period, which is given by $q_{1s} = F'(k_{1s}^T)$ (where $q_0 = q_{2s} = 1$). Furthermore, in equilibrium, it will be never optimal to pay dividends in $t = 0$ and $t = 1$ and, hence, I omit those choice variables from the set-up without loss of generality. Since I assume that banker i does not have an access to a commitment technology, I solve the model backwards.²²

The actions in reverse order are the following. In $t = 2$, all bankers produce and pay out all the profits as dividends to the consumers. At the end of $t = 1$, banker i maximizes the dividend payment in the last period by choosing $\{k_{1s}^i, d_{2s}^i\}$ and taking as given the state variables $\{B_s^i, k_0^i, d_{1s}^i\}$. Banker i maximizes

$$\max_{k_{1s}^i, d_{2s}^i} (A + 1 - \gamma) k_{1s}^i - d_{2s}^i$$

subject to the collateral constraint in $t = 1$

$$d_{2s}^i \leq \theta (1 - \gamma) k_{1s}^i \quad [\lambda_{2s}^i] \quad (1.3.1)$$

where the Lagrangians are given in square brackets. Banker i also takes into account the period one budget constraint

$$k_{1s}^i F'(k_{1s}^T) + d_{1s}^i \leq (F'(k_{1s}^T) + a_{1s} - \gamma) k_0^i + B_s^i + d_{2s}^i \quad [z_{1s}^{i,1}] \quad (1.3.2)$$

where $B_h^i = 0$ since I assumed bail-outs are prohibitively costly if there is no crisis.²³

²²There will be a time inconsistency problem since when banks are large, they internalize the fact that their period one investment decision will affect the tightness of their period zero borrowing constraint (there will be no time inconsistency in the case with a continuum of banks). For more details see Davila (2011).

²³The implicit assumption is that there is a large fixed cost of providing a transfer of money from the consumers to bankers if there is no fire sale (no crisis).

At the beginning of $t = 1$, first, banker i repays the promised debt d_{1s}^i to the consumers. After that, if the low state is realized, the policy maker chooses B_l^i given the state variables $\{k_0^i, d_{1s}^i\}$ and the optimal decision of banker i at the end of $t = 1$. He also takes into account that his choice of B_l^i affects the choices banker i will make at the end of $t = 1$. Given that consumers are risk neutral, the objective function of a benevolent policy maker in $t = 1$ in the low state is to maximize last period's total output.

$$\begin{aligned} & \max_{k_{1l}^i, B_l^i} 2e + F(k_{1l}^T) - F'(k_{1l}^T) k_{1l}^T - \delta(B_l) + d_{1l} \\ & + \sum_{i=1}^N \frac{1}{N} \left[\begin{aligned} & (A + 1 - \gamma) k_{1l}^i - d_{2l}^i - B_l^i + \\ & z_{1l}^{i,1,P} ((F'(k_{1l}^T) + a_{1l} - \gamma) k_0^i + B_l^i + d_{2s}^i - k_{1l}^i F'(k_{1l}^T) - d_{1l}^i) \end{aligned} \right] \end{aligned}$$

$(A + 1 - \gamma) k_{1l}^i - d_{2l}^i$ are the dividends paid by banker i to the equity owners (consumers) in $t = 2$ in the low state, $F(k_{1l}^T) - F'(k_{1l}^T) k_{1l}^T$ are the profits of the consumers from operating their production technology if a fire sale is present. d_{1l} is the period one payment by the bankers to the consumers. $\delta(B_l) + B_l$ is the cost of the bail out — direct cost plus the deadweight loss from taxing.²⁴

At the end of $t = 0$, banker i optimizes the expected value of dividends paid to the consumers in $t = 2$, taking into account his future optimal actions and the formula for the optimal bail-out in the crisis state. The period zero objective function of banker i is given by

$$\max_{k_0^i, d_{1s}^i} \sum_s \pi_s [(A + 1 - \gamma) k_{1s}^i - d_{2s}^i]$$

subject to the budget constraints in $t = 1$ given by equation 1.3.2, the period zero Lagrangian of which is $\pi_s z_{1s}^{i,0}$. Also banker i takes into account the period zero budget constraint

$$k_0^i \leq \sum_s \pi_s d_{1s}^i + n \quad [z_0^i] \quad (1.3.3)$$

The optimization problem is also subject to the $t = 0$ collateral constraints.

$$d_{1s}^i \leq \theta (F'(k_{1s}^T) - \gamma) k_0^i \quad [\pi_s \lambda_{1s}^i] \quad (1.3.4)$$

For detailed derivations see Appendix, Section A.1.2. From now on the $*$ denotes the optimal allocation from the decentralized equilibrium.

²⁴I also assume that the endowment of the consumer, e , is large enough so that $B_l < e + d_{1l} - p_{2l} d_{2l}$.

Proposition 1.3.1 *Given Assumptions 1.1-1.7 and the Assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, considering a symmetric equilibrium with an ex-post optimal bail-out and no ex-ante regulation: (i) There is no fire sale in the high state, $q_{1h} = 1$ and there is a fire sale in the low state, $q_{1l} < 1$. (ii) Given the additional Assumption 1.8 provided in the Appendix, (required only for the $N < \infty$ case), the equilibrium is unique and exists and is one of the following types: Type 1) $z_0^* = z_{1l}^* > z_{1h}^*$; ($\lambda_{1l}^* = 0, \lambda_{1h}^* > 0$) (interior equilibrium) Type 2) $z_0^* > z_{1s}^*$; ($\lambda_{1s}^* = 0$) (corner equilibrium where the banker borrows to the maximum in $t = 0$) where*

$$z_0^* = \sum_s \pi_s \left[z_{1s}^* \left(F'(k_{1s}^T) + a_{1s} - \gamma + \frac{1}{N} F''(k_{1s}^T) k_{1s}^T + \frac{1}{\delta''(B_s)N} \frac{\partial z_{1s}^{1,P}}{\partial k_{1s}^T} \right) + \lambda_{1s}^* \theta \left(F'(k_{1s}^T) - \gamma + \frac{1}{N} F''(k_{1s}^T) k_0 \right) \right] \quad (1.3.5)$$

$$z_{1s}^* = \frac{A + (1 - \theta)(1 - \gamma) - \lambda_{1s}^* \theta \frac{1}{N} F''(k_{1s}^T) k_0}{\frac{1}{N} F''(k_{1s}^T) k_{1s}^T + \frac{1}{\delta''(B_s)N} \frac{\partial z_{1s}^{1,P}}{\partial k_{1s}^T} + F'(k_{1s}^T) - \theta(1 - \gamma)} \quad (1.3.6)$$

and $z_{1s}^* = z_{1s}^{0,*}$. The optimal bail-out is pinned down by

$$1 + \delta'(B_l) = z_{1l}^{1,P} = \frac{F''(k_{1l}^T) k_{1l}^T + A + (1 - \theta)(1 - \gamma)}{F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma)} \quad (1.3.7)$$

where $\frac{\partial B_s^i}{\partial k_0^i} = -\frac{\partial B_s^i}{\partial k_{1s}^i} = \frac{1}{\delta''(B_s)N} \frac{\partial z_{1s}^{1,P}}{\partial k_{1s}^T}$, and the first order conditions with respect to d_{2s} and d_{1s} imply $\lambda_{2s}^* > 0, \lambda_{1s}^* = z_0^* - z_{1s}^* \geq 0$.

Proof of Proposition 1.3.1. See Appendix, Section A.1.4. ■

The key variables that characterize the equilibrium type are z_{1s}^* and z_0^* . z_0^* is the period zero marginal value of an extra unit of the consumption good (which, from now on, I will refer to as an extra dollar) in the hands of the banker as perceived by the banker. z_{1s}^* is the scaled marginal value of an extra dollar in $t = 1$, state s , in the hands of the banker as perceived by the banker. When the banker decides optimally whether to keep an extra dollar in $t = 0$ or to "transfer" it to $t = 1$, state s , using a state contingent contract, the relevant variables to compare are z_0^* and z_{1s}^* . If the banker saves/"transfers" an extra dollar from $t = 0$ to period 1, state s , he will get $\frac{1}{\pi_s}$ units of the consumption good in $t = 1$, state s , and his ex-ante welfare will increase by $(\pi_s z_{1s}^*) \frac{1}{\pi_s} = z_{1s}^*$. If the banker keeps the dollar in $t = 0$, his ex-ante welfare will improve by z_0^* . Throughout the rest of the paper, I will refer to z_{1s}^* and z_0^* as the marginal value of wealth in the hands of the banker as perceived by the banker in $t = 1$, state s , and in $t = 0$ respectively.

Let us analyze the first order conditions in more detail. One can prove that $z_{1s}^* > 1$ and $z_0^* > 1$. This implies that in $t = 0$ and in $t = 1$, the banker optimally does not pay dividends to the consumer whose

marginal value of wealth is one. This confirms the assumption made when solving the problem that dividends will be optimally paid out only in $t = 2$. Also $z_{1s}^* > 1$ implies that the banker wants to transfer the maximum amount of resources from period two to period one since in the last period the marginal value of wealth is simply one (equal to the marginal utility of consumption of the consumers). As a result, the period one borrowing constraints will always bind, implying that $\lambda_{2s}^* > 0$ and $d_{2s} = \theta(1 - \gamma)k_{1s}$. If the equilibrium is interior, then $z_0^* = z_{1l}^* > z_{1h}^*$, which implies that the banker values wealth more in period zero and in the crisis state than he values wealth in the high state in $t = 1$. Therefore, in period zero, the banker borrows to the maximum against the high state which implies $\lambda_{1h}^* > 0$.

The economic intuition behind the first order conditions is the following. The first order condition of the policy maker with respect to the bail-out at the beginning of $t = 1$, equation 1.3.7, determines the optimal bail-out, B_l , by equating the marginal cost to the marginal benefit of the bail-out. The marginal cost of the bail-out is simply the direct transfer of one dollar from the consumer to the banker plus the marginal increase of the deadweight loss from the bail-out, $\delta'(B_l)$. The marginal benefit of the bail-out is the marginal value of an extra dollar in the hands of the bankers in the crisis state as perceived by the policy maker, $z_{1l}^{1,P}(k_{1l}^T)$. From equation 1.3.7, it is clear that only the aggregate bail-out is pinned down, B_l , and not the bank specific one, B_l^i . In order to solve the model, I assume that the equilibrium is symmetric where the government gives the same bail-out to each bank, $B_l = B_l^i$, and every banker internalizes that when making decisions in period $t = 0$.²⁵

By totally differentiating equation 1.3.7 with respect to $k_{1l}^{i,T}$, one can derive how banker i perceives his individual fire sale to affect the targeted bail-out he receives, $\frac{\partial B_l^i}{\partial k_{1l}^{i,T}}$, which will be an important term that enters the first order conditions with respect to k_0^i and k_{1l}^i .

$$\frac{\partial B_l^i}{\partial k_{1l}^{i,T}} = \begin{cases} \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}(k_{1l}^T)}{\partial k_{1l}^T} > 0 \text{ if } N < \infty \text{ and } \chi > 0 \\ 0 \text{ if } N \rightarrow \infty \text{ or } \chi = 0 \end{cases}$$

When banks are not infinitesimally small and the country has some fiscal capacity, $N < \infty$ or $\chi > 0$, banker i partially internalizes the fact that the larger his individual fire sale is, the larger the optimal bail-out is, $\frac{\partial B_l^i}{\partial k_{1l}^{i,T}} > 0$. The reason is the following. If the fire sale is large, the price of capital in $t = 1$, state l , is

²⁵In this model, with linear production technology of the bankers, the policy maker is indifferent whether to give the bail-out money to Bank of America which takes over Merrill Lynch (the bank that needs the bail-out) or he gives the money directly to Merrill Lynch. Optimally, the policy maker wants to achieve an aggregate fire sale of k_{1l}^T which is determined by equation 1.3.7. For a proof of this result, see Appendix, Section A.1.2. Of course, the ex-post bail-out design affects the ex-ante incentives of banks. Acharya, Shin, and Yorulmazer (2011) and Nosal and Ordóñez (2013) are two interesting papers that address the question of what is the optimal ex-post bail-out design that minimizes moral hazard ex-ante.

In this model I focus on an environment where the policy maker has a sufficient number of ex-ante instruments to correct for the moral hazard (he can replicate the constrained Central Planner's allocation). Therefore, the ex-post bail-out design is not crucial for the results.

lower and the transfer of resources from the more productive sector — bankers — to the less productive sector — consumers — is larger. As a result, when the fire sale is larger, the policy maker values an extra dollar in the hands of the banker in a crisis by more, $\frac{\partial z_{1l}^{1,P}(k_{1l}^T)}{\partial k_{1l}^T} > 0$, and optimally provides a larger bail-out. However, for countries with a large number of banks, for a given level of the fire sale, the perceived impact of the individual fire sale on the aggregate fire sale is smaller, which is captured by the $\frac{1}{N}$ term. Finally, the bail-out received is smaller, for a given level of the fire sale, if the country is more fiscally constrained, which is captured by the fact that $\delta''(B_l^i)$ is positive and increases when χ decreases. In the limit case with a continuum of banks, $N \rightarrow \infty$, $\frac{\partial B_l^i}{\partial k_{1l}^{i,T}} = 0$ because $z_{1l}^{1,P}$ is a function only of the aggregate fire sale and bankers are too small to affect the aggregate fire sale and, hence, the bail-out they receive. (This result will be crucial to prove later on that in the case with a continuum of banks there is no moral hazard while there will be moral hazard if banks are large.) Also if the country has no fiscal capacity, $\chi = 0$, the bail-out will be zero and, hence, $\frac{\partial B_l^i}{\partial k_{1l}^{i,T}} = 0$.

Equation 1.3.6 is the first order condition with respect to k_{1s}^i (from the period zero optimization problem), which pins down the marginal value of wealth in the hands of the bankers in period one as perceived by the banker, z_{1s}^* . In equilibrium, z_{1s}^* is equal to the marginal benefit of k_{1s} over the "effective" marginal cost of purchasing an extra unit of k_{1s} . The direct marginal cost of capital is the price of capital $q_{1s} = F'(k_{1s}^T)$ which is lowered by the fact that the banker can lever against the capital — captured by the term $-\theta(1-\gamma)$. The indirect marginal cost is given by the term $\frac{1}{N}F''(k_{1s}^T)k_{1s}^T + \frac{1}{\delta''(B_s)N}\frac{\partial z_{1s}^{1,P}}{\partial k_{1s}^T}$ and is relevant only if the bank is not infinitesimally small ($N < \infty$) and there is a fire sale. The indirect cost includes a monopolistic effect, $\frac{1}{N}F''(k_{1s}^T)k_{1s}^T < 0$, which makes the "effective" cost of period one capital lower because the banker realizes that an extra unit of period one capital will increase the per unit price of the fire-sold capital (for a given k_0), k_{1s}^T , if any. In addition, if the bank is large and there is a fire sale, it also realizes that an extra k_{1s} will decrease the marginal bail-out received by $\frac{\partial B_s^i}{\partial k_{1s}^T} = -\frac{1}{\delta''(B_s)N}\frac{\partial z_{1s}^{1,P}}{\partial k_{1s}^T} < 0$ ²⁶ because the fire sale will be smaller. This last effect will increase the "effective" marginal cost of period one capital. The marginal benefit of an extra dollar invested is the cash flow received in $t = 2$, A , plus the resale value of capital of one minus the refinancing cost and minus the debt payment $(1-\theta)(1-\gamma)$. If the Lagrangian on the period zero borrowing constraint in the crisis state is binding, $\lambda_{1s}^* > 0$, the extra unit of capital in period one will increase the resale value of period zero capital and relax the period zero constraint, which is captured by the $-\lambda_{1s}^*\theta\frac{1}{N}F''(k_{1s}^T)k_0$ term.²⁷ This will increase the marginal benefit of k_{1s} .

The first order condition with respect to k_0 , given by equation 1.3.5, pins down the marginal value of

²⁶ $\frac{\partial B_s}{\partial k_{1s}^T}$ is a partial derivative for a given k_0^i

²⁷ In the interior equilibrium case (Type 1 equilibrium), which is the more interesting case, $\lambda_{1l}^* = 0$.

wealth in period zero as perceived by the banker, z_0^* . Since the price of period zero capital is one, an extra dollar in $t = 0$ implies an extra unit of capital purchased in $t = 0$. Therefore, the marginal value of wealth in period zero z_0^* , in equilibrium, is equal to the marginal benefit of k_0 . In $t = 1$, state s , the return from an extra dollar of period zero investment is the cash flow a_{1s} plus the resale price, $q_{1s} = F'(k_{1s}^T)$, minus the refinancing cost, γ . In addition, if the banker is not infinitesimally small, he internalizes the fact that the "effective" marginal return to k_0 is smaller, because an extra k_0 leads to a larger fire sale and a lower price of the fire-sold capital. Hence the return will decrease by $\frac{1}{N}F''(k_{1s}^T)k_{1s}^T < 0$ (a monopolistic effect).²⁸ However, higher k_0 will also increase the perceived bail-out received by increasing the fire sale, which is captured by the term $\frac{\partial B_s^i}{\partial k_0^i} = \frac{1}{\delta''(B_s)N} \frac{\partial z_{1s}^{1,P}}{\partial k_{1s}^T} > 0$ (partial derivative given k_{1s}). This would increase the "effective" return to an extra dollar invested in period zero. Notice that the banker internalizes only the benefit but not the cost of the bail-out. The "effective" period one return on k_0 is re-invested, which is why it is multiplied by the marginal value of wealth in $t = 1$ state s , z_{1s}^* . If the period zero collateral constraint is binding, an extra k_0 has the additional benefit of relaxing the borrowing constraint, which is why the marginal benefit of k_0 also includes the term $\lambda_{1s}^* \theta (F'(k_{1s}^T) - \gamma + \frac{1}{N}F''(k_{1s}^T)k_0)$.

Graphical Proof of Existence and Uniqueness

In order to solve for the optimal allocation, first, I solve for the optimal amount of period zero investment, k_0^* . I will provide intuition for the proof of existence and uniqueness using a graphical approach of how k_0^* is determined. In subsequent sections, I will build on Figure 1.3.1 to prove that there is overinvestment if the policy maker does not have an access to any regulatory instruments besides the ex-post bail-out. I will also use a figure similar to Figure 1.3.1 to prove the key result of this paper — that smaller fiscal capacity optimally implies a larger ex-ante minimum bank capital ratio conditional on sufficient number of instruments to replicate the constrained Central Planner's allocation.

Define the following function of k_0

$$\psi^*(k_0) = z_{1l}^*(k_0) - z_0^*(k_0) \text{ where } k_0 \in [\tilde{k}_0, k_0^{\max}]$$

$[\tilde{k}_0, k_0^{\max}]$ is the relevant range for k_0 if the equilibrium is the interior equilibrium of Type 1 (See the proof of Proposition 1.3.1 for details regarding how to derive the relevant range) and $z_{1l}^*(k_0)$ and $z_0^*(k_0)$ are the marginal values of wealth in period zero and in the low state in period one if the equilibrium is of Type

²⁸In the proof of Proposition 1.3.1 in the Appendix, I show that the larger period zero investment is, k_0 , the larger the fire sale is, k_{1l}^T , and there is a one-to-one mapping between k_0 and k_{1l}^T .

1. If the equilibrium is of Type 1 (interior equilibrium), k_0^* will be determined by $\psi^*(k_0^*) = 0$ and if the equilibrium is of Type 2 (corner equilibrium) $k_0^* = k_0^{\max}$ (the bank will borrow to the maximum in $t = 0$).²⁹

Let us focus on the interior equilibrium of Type 1 which will be the more interesting case. k_0^* is pinned down using the first order condition with respect to d_{1l}^i , which is similar to an Euler equation and given by $z_{1l}^*(k_0^*) = z_0^*(k_0^*)$. In equilibrium, the banker is indifferent between investing an extra dollar in period zero and saving the extra dollar for the crisis state using a state contingent contract. If the banker invests the extra dollar in period zero, his ex-ante welfare increases by the marginal benefit of an extra k_0 , $z_0^*(k_0^*)$. If he saves the extra dollar towards the crisis state and invests it then, his ex-ante welfare increases by $z_{1l}^*(k_0^*)$. Figure 1.3.1 depicts $\psi^*(k_0)$.³⁰

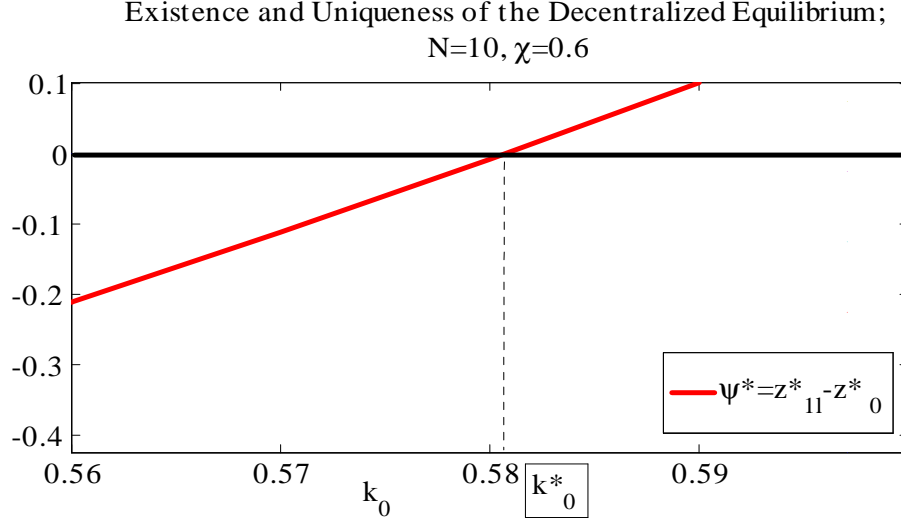


Figure 1.3.1

First, notice that $\psi^*(k_0)$ is a strictly increasing function of k_0 because the marginal value of wealth in the crisis state, as perceived by the banker, $\frac{\partial z_{1l}^*}{\partial k_0} > 0$, increases with period zero investment while the marginal value of wealth in period zero decreases with period zero investment, $\frac{\partial z_0^*}{\partial k_0} < 0$. Let us start with $\frac{\partial z_{1l}^*}{\partial k_0} > 0$. In the case with a continuum of banks, $N \rightarrow \infty$, $\frac{\partial z_{1l}^*}{\partial k_0} > 0$ because a larger k_0 increases the fire sale which lowers the price of capital $q_{1l} = F'(k_{1l}^T)$. As a result, an extra dollar is more valuable in the crisis state since it can purchase more units of capital. If banks are large, $N < \infty$, in order for $\frac{\partial z_{1l}^*}{\partial k_0} > 0$, Assumption 1.8 is required as well. Assumption 1.8 guarantees that the "effective" cost of capital in the crisis state decreases

²⁹If $\psi^*(k_0) < 0$ for all $k_0 \in [\tilde{k}_0, k_0^{\max}]$, the equilibrium is of Type 2 and in Proposition 1.3.1 in the Appendix, I prove that it will be never the case that $\psi^*(k_0) > 0$ for all $k_0 \in [\tilde{k}_0, k_0^{\max}]$.

³⁰The functional forms used for $\delta(\cdot)$ and $F(\cdot)$ in the Figure are given in the beginning of the Appendix. The parameters used to produce Figure 1.3.1 are $\gamma = 0.7, \alpha = 0.8, A = 1, a_{1h} = 1.5, a_{1l} = 0, \pi_h = .55, n = .5, \theta = 1, \eta = 1.5, N = 10$

as k_0 increases, because the direct effect of the price decrease as k_0 increases is not offset by the fact that the perceived marginal bail-out received increases with k_0 , $\frac{\partial^2 B_l^i}{\partial k_{1l}^{i,T} \partial k_0} > 0$.³¹

In addition to $\frac{\partial z_{1l}^*}{\partial k_0} > 0$, also $\frac{\partial z_0^*}{\partial k_0} < 0$. In equilibrium z_0^* is equal to the marginal benefit of an extra k_0 . An extra k_0 will lead to a larger fire sale and to a lower re-sale price of k_0 in the crisis state, q_{1l} , thereby lowering the marginal benefit of an extra dollar invested in $t = 0$. In the case of a finite number of banks, $N < \infty$, Assumption 1.8 guarantees that this direct effect dominates the indirect effect, where larger k_0 leads to a larger fire-sale in the crisis state and larger perceived marginal bail-out, $\frac{\partial^2 B_l^i}{\partial k_{1l}^{i,T} \partial k_0} > 0$, which increases the marginal benefit of an extra dollar invested in $t = 0$. Therefore, as k_0 increases, z_{1l}^* increases and z_0^* decreases, leading to $\psi^{*'}(k_0) > 0$. As a result, $\psi^*(k_0)$ will cross the zero line at most once.

1.3.2 Constrained Central Planner's Problem Without Commitment

In this section I solve for the constrained Central Planner's problem without commitment. The constrained Central Planner optimizes the welfare of the consumers who are also the owners of the banks. The Central Planner faces exactly the same constraints as the banker in the decentralized equilibrium — the borrowing constraints plus the first order conditions of the consumer. Setting up the constrained Central Planner's problem in such a way is equivalent to allowing the Central Planner to affect only the actions of the banker but he cannot directly transfer resources from the consumer to the banker unless he uses the bail-out instrument which is costly. Given that markets are incomplete due to the borrowing constraints, if we were to allow the Central Planner to circumvent the first order conditions of the consumer, he could directly transfer resources from consumers to bankers without incurring any cost, which would make the problem trivial and unrealistic.

Essentially, the Central Planner chooses the investment and borrowing decision of every banker i , where he will take into account any externalities this decision imposes on the rest of the bankers and on consumers. Given that the equilibrium is symmetric, this maps into a problem where the Central Planner chooses aggregate variables, taking into account that his actions affect prices and, in particular, the price of capital in $t = 1$, $q_{1s} = F'(k_{1s}^T)$. The Central Planner also takes into account the rest of the equilibrium prices given by $q_0 = q_{2s} = 1$, $p_{1s} = p_{2s} = 1$. In this section, I preserve the assumption that the bail-out will be possible only if there is a fire sale.

³¹Intuitively, the direct effect is that an extra k_0 leads to a larger fire-sale which makes an extra dollar in the crisis state *more* valuable because it is cheaper to purchase an extra dollar of capital, and the indirect effect is that it also makes an extra dollar in the crisis state *less* valuable because this extra dollar invested in k_{1l} will decrease the fire-sale and hence will decrease the marginal bail-out received. Assumption 1.8 guarantees that the direct effect on the price of capital is the one that dominates.

Solving the problem backwards, in $t = 1$, the Central Planner maximizes the welfare of the consumers given the state variables $\{k_0, d_{1s}\}$, which, in this environment with risk neutral consumers, coincides with maximizing aggregate output. The period one optimization problem is given by

$$\max_{B_s, k_{1s}, d_{2s}} 2e + F(k_{1s}^T) - F'(k_{1s}^T) k_{1s}^T + d_{1s} - (B_s + \delta(B_s)) + (A + 1 - \gamma) k_{1s} - d_{2s}$$

subject to the collateral constraint in period one, equation 1.3.1, with a Lagrangian given by λ_{2s}^{CP} , and subject to the period one budget constraint, equation 1.3.2, with a Lagrangian given by $z_{1s}^{1,CP}$. Notice that the Central Planner internalizes the cost of the bail-out, $-(B_s + \delta(B_s))$. He also takes into account that a larger aggregate fire sale improves the welfare of the consumer via the profits from operating the consumer's production technology given by $F(k_{1s}^T) - F'(k_{1s}^T) k_{1s}^T$. As a result, the Central Planner will internalize the externalities imposed by a single banker on the consumers. The Central Planner also internalizes the fact that the actions of a single banker affect the price of fire sold capital in the crisis state which enters the budget constraint of the banker. This will be the mechanism through which the Central Planner internalizes the externalities that a single banker will impose on the rest of the bankers. The latter mechanism will be at the heart of the pecuniary externality.

In $t = 0$, the Central Planner chooses $\{k_0, d_{1s}\}$, taking into account his future optimal actions, in order to optimize the following ex-ante welfare function

$$\max_{k_0, d_{1s}} 3e + \sum \pi_s [F(k_{1s}^T) - F'(k_{1s}^T) k_{1s}^T - (B_s + \delta(B_s)) + (A + 1 - \gamma) k_{1s} - d_{2s}]$$

The period zero optimization problem is subject to the budget constraint in $t = 1$, equation 1.3.2, with a Lagrangian given by $\pi_s z_{1s}^{0,CP}$ and the budget constraint in $t = 0$, equation 1.3.3, where the Lagrangian is z_0^{CP} . The Central Planner also takes into account the period zero collateral constraint given by equation 1.3.4 and the Lagrangian is $\pi_s \lambda_{1s}^{CP}$. For details on the set-up and the solution see Appendix, Section A.1.3.

Proposition 1.3.2 (i) *Given Assumptions 1.1-1.7 and the assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, there is never a fire sale in the high state, $q_{1h} = 1$ and there is a fire sale in the low state, $q_{1l} < 1$.* (ii) *The equilibrium of the constrained Central Planner's problem exists and is unique and is one of the following types: Type 1) $z_0^{CP} = z_{1l}^{CP} > z_{1h}^{CP}$ (interior equilibrium); Type 2) $z_0^{CP} > z_{1s}^{CP}$ (corner equilibrium where the banker borrows to the maximum in $t = 0$) where $z_{1s}^{CP} = z_{1s}^{0,CP}$. The optimal bail-out is determined by*

$$1 + \delta'(B_l) = z_{1l}^{1,CP} \quad (1.3.8)$$

(iii) If also Assumption 1.9 is satisfied (provided in the Appendix, Section Assumption 1.8), the only possible equilibrium is the interior equilibrium of Type 1 where

$$\sum \pi_s \left(-F''(k_{1s}^T) k_{1s}^T + z_{1s}^{CP} (F'(k_{1s}^T) + a_{1s} - \gamma + F''(k_{1s}^T) k_{1s}^T) \right) + \pi_h \lambda_{1h}^{CP} \theta [1 - \gamma] = z_0^{CP} \quad (1.3.9)$$

$$z_{1h}^{CP} = \frac{A + (1 - \theta)(1 - \gamma)}{1 - \theta(1 - \gamma)} \quad (1.3.10)$$

$$z_{1l}^{1,CP} = z_{1l}^{CP} = \frac{F''(k_{1l}^T) k_{1l}^T + A + (1 - \theta)(1 - \gamma)}{F'(k_{1l}^T) - \theta(1 - \gamma) + F''(k_{1l}^T) k_{1l}^T} \quad (1.3.11)$$

and the first order conditions with respect to d_{2s} and d_{1s} imply $\lambda_{2s}^{CP} > 0, \lambda_{1l}^{CP} = 0, \lambda_{1h}^{CP} = z_0^{CP} - z_{1h}^{CP} \geq 0$.

Proof of Proposition Assumption 1.8. See Appendix, Section Assumption 1.8. ■

Similarly to the decentralized equilibrium, one can prove that $z_{1s}^{CP} > 1$ and $z_0^{CP} > 1$. The equilibria types, which are classified based on the borrowing contract between the banker and the consumer, are the same as the decentralized equilibria types. If the equilibrium is an interior equilibrium, the Central Planner borrows to the maximum against the value of last period capital, $\lambda_{2s}^{CP} > 0$. Furthermore, in $t = 0$, he borrows first against the high state and only then against the low state in $t = 0$, i.e. $\lambda_{1l}^{CP} = 0$ and $\lambda_{1h}^{CP} > 0$. The Central Planner perceives an extra dollar to be more valuable in the crisis state rather than in the non-crisis state due to the presence of a fire sale which increases the inefficient transfer of resources from the more productive sector — bankers — to the less productive sector — consumers.

Regarding proving existence and uniqueness of the constrained Central Planner's allocation, one can use a similar graphical approach as in the case of the decentralized equilibrium. Define the following function

$$\psi^{CP}(k_0) = z_{1l}^{CP}(k_0) - z_0^{CP}(k_0)$$

where $z_0^{CP}(k_0)$ and $z_{1l}^{CP}(k_0)$ are the marginal values of wealth in the hands of the bankers as perceived by the Central Planner in period zero and in the low state in period one if the equilibrium is of Type 1. If the equilibrium is of Type 1, k_0^{CP} will be determined by $\psi^{CP}(k_0^{CP}) = 0$ and if the equilibrium is of Type 2, $k_0^{CP} = k_0^{\max}$ (the bank will borrow to the maximum in $t = 0$). In order to prove existence and uniqueness it will be sufficient to prove that $\psi^{CP'}(k_0) > 0$. As before, larger k_0 implies larger fire sale, k_{1l}^T and lower price of capital. As a result, more wealth is transferred from the more productive to the less productive sector and, hence, the marginal value of wealth in the crisis state as perceived by the Central Planner, z_{1l}^{CP} , is larger. At the same time, z_0^{CP} , which in equilibrium equals the marginal benefit of an extra k_0 as perceived by the Central Planner, decreases with k_0 . The reason is that a larger k_0 implies a larger fire sale and larger

inefficient transfer of resources. Hence the marginal benefit of k_0 , z_0^{CP} , decreases with k_0 . (For details on the derivations see the proof of Proposition 1.3.2).

1.4 Overinvestment

In this sub-section, I compare the constrained Central Planner's allocation and the decentralized equilibrium with no ex-ante regulation. I prove that when we start the economy in normal times, which is the case with this model (i.e. no fire sale in $t = 0$), there is ex-ante overinvestment.³² If $N \rightarrow \infty$, the overinvestment is due to future pecuniary externalities and, if $N < \infty$, it is due to both future pecuniary externalities and moral hazard.

Before proving overinvestment, first I prove Corollary 1.4.1, which states that the marginal value of wealth in the hands of the banker in the crisis state, as perceived by the Central Planner, is larger than the marginal value of wealth in the hands of the banker in the crisis state, as perceived by the banker in the decentralized equilibrium, $z_{1l}^{CP} > z_{1l}^*$. This result will be the main component to the proof that there is ex-ante overinvestment in this model due to future pecuniary externalities.

Corollary 1.4.1 *Conditional on Assumptions 1.1-1.7 and the assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, also conditional on an interior equilibrium for the Central Planner (Assumption 1.9 is satisfied) and given Assumption 1.10, (a sufficient and necessary condition only for the $N < \infty$ case)*

$$\frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} z_{1l}^{CP} > \left(\left(1 - \frac{1}{N} \right) z_{1l}^{CP} - 1 \right) F''(k_{1l}^T) k_{1l}^T \quad \text{Assumption 1.10}$$

the Central Planner values an extra dollar in the hands of the banker in the crisis state by more than the banker in the decentralized equilibrium does, for a given k_0 .

$$z_{1l}^{CP} > z_{1l}^*$$

Proof of Corollary 1.4.1. See Appendix, Section **Assumption 1.9.** ■

Having proved Corollary 1.4.1, I proceed to prove overinvestment.

³²In an extension in an older version of the paper, I showed that if the economy starts in a crisis state with a fire sale, contemporaneous pecuniary externalities can potentially lead to underinvestment and not overinvestment. The reason why this is the case is similar to the result in He and Kondor (2012).

Proposition 1.4.2 *Conditional on Assumptions 1.1-1.8 and 1.10 and the Assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, comparing the constrained Central Planner's allocation and the decentralized equilibrium with no ex-ante regulation, there is always overinvestment, $k_0^{CP} < k_0^*$, if the equilibrium is of Type 1 for the Central Planner (interior equilibrium) and there is no overinvestment, $k_0^{CP} = k_0^*$, if the equilibrium is of Type 2 for the Central Planner (corner equilibrium).*

Proof of Proposition 1.4.2. Since I already proved in the previous sections that given the assumptions made, $\psi^{CP_I}(k_0) > 0$ and $\psi^{*I}(k_0) > 0$, in order to prove that there is overinvestment, it is sufficient to show that as long as both the Central Planner's equilibrium and the decentralized equilibrium are of Type 1, $\psi^{CP}(k_0) > \psi^*(k_0)$ (see Figure 2 below). One can re-write $\psi^{CP}(k_0) - \psi^*(k_0)$ as

$$\psi^{CP}(k_0) - \psi^*(k_0) = \frac{(z_{1l}^{CP}(k_0) - z_{1l}^*(k_0))(1 - \theta[1 - \gamma] + \pi_l[\gamma - a_{1l}])}{(1 - \pi_h\theta[1 - \gamma])} > 0 \quad (1.4.1)$$

where the inequality follows from Assumption 1.3 and Corollary 1.4.1. It is clear that if the equilibrium is of Type 1 for the Central Planner (Assumption 1.9 is satisfied) and Type 2 for the banker, then there is overinvestment since Type 2 equilibrium implies that the banker will borrow to the maximum ex-ante. Also if the equilibrium of the Central Planner is of type 2, there will be no overinvestment and one can easily show that the equilibrium will be of Type 2 for the banker as well. ■

Figure 1.4 depicts $\psi^*(k_0)$ and $\psi^{CP}(k_0)$, using the same parametrization as in Figure 1.3.1.

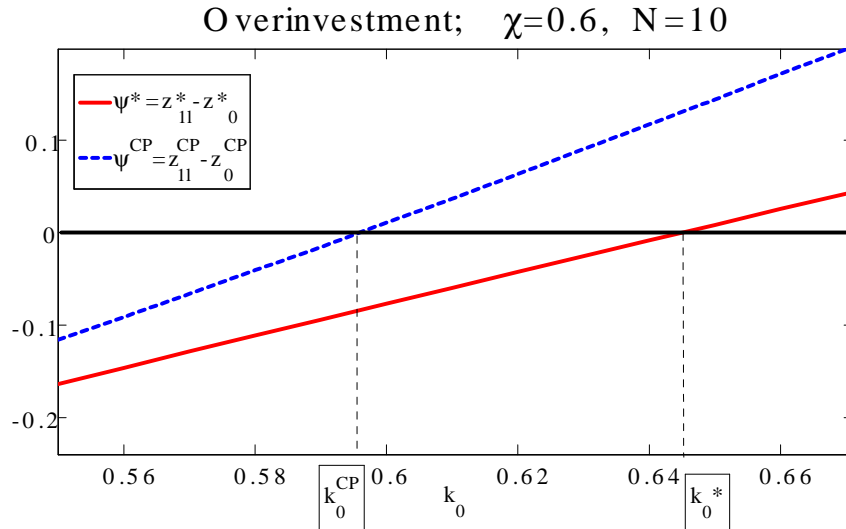


Figure 1.4

From equation 1.4.1 one can see that proving overinvestment is equivalent to proving that $z_{1l}^{CP}(k_0) > z_{1l}^*(k_0)$ where the equilibrium is of Type 1 for the Central Planner and for the banker in the decentralized

equilibrium. There are two reasons why there is overinvestment in this model — future pecuniary externalities and moral hazard.

Future Pecuniary Externality

In order to isolate the pecuniary externality channel, let us first consider the case of a country with no fiscal capacity, $\chi = 0$, and hence, no bail-out $B_l = 0$. The lack of bail-out implies that the difference between $z_{1l}^{CP}(k_0)$ and $z_{1l}^*(k_0)$ is only due to the pecuniary externalities. The first reason why $z_{1l}^{CP}(k_0)$ differs from $z_{1l}^*(k_0)$ is that the Central Planner internalizes the fact that an extra dollar in the crisis state in the hands of a single banker will decrease the fire sale, which will relax the budget constraints of the other bankers (captured by the $F''(k_{1l}^T) k_{1l}^T < 0$ term in the denominator of $z_{1l}^{CP}(k_0)$). In contrast, the banker only partially internalizes his impact on prices if $N > 1$ (captured by the $\frac{1}{N} F''(k_{1l}^T) k_{1l}^T < 0$ term in the denominator of $z_{1l}^*(k_0)$). This effect pushes $z_{1l}^{CP}(k_0)$ to be larger than $z_{1l}^*(k_0)$. The Central Planner, unlike the banker, also internalizes the fact that an extra dollar in the hands of the banker in the crisis state implies a smaller fire sale and lower profits for the consumer (which is captured by the $F''(k_{1l}^T) k_{1l}^T$ term in the numerator of $z_{1l}^{CP}(k_0)$) which pushes $z_{1l}^{CP}(k_0)$ to be lower than $z_{1l}^*(k_0)$.³³ Assumption 1.10 is a sufficient and necessary condition for the former effect to dominate the latter. For example, if $N \rightarrow \infty$, Assumption 1.10 is always satisfied while if $N = 1$, it will not be satisfied if $\chi = 0$.³⁴ The economic intuition behind the pecuniary externality and the overinvestment is that, the Central Planner, unlike the banker, internalizes the fact that an extra dollar of capital in the hands of the bankers during a crisis implies that less capital will be transferred from the more productive users of capital — bankers — to the less productive users of capital — consumers. Since the Central Planner values wealth in the crisis state more than the banker and higher k_0 leads to a lower bank net worth in the crisis state, the Central Planner optimally chooses a lower k_0 than the banker in the decentralized equilibrium (as can be seen on Figure 2).³⁵

If $N \rightarrow \infty$, which is the continuum of banks case, banks take the price of capital as given, and the pecuniary externality is the strongest. The smaller N is, the smaller the pecuniary externality is and, the difference between $z_{1l}^{CP}(k_0)$ and $z_{1l}^*(k_0)$ shrinks. Finally, notice that if a country has a continuum of banks and no fiscal capacity ($\chi = 0$ and $N \rightarrow \infty$), $z_{1l}^{CP}(k_0) - z_{1l}^*(k_0)$ coincides with the $z_{1l}^{CP}(k_0) - z_{1l}^*(k_0)$ of a country with a continuum of banks and any level of fiscal capacity ($N \rightarrow \infty$ for any χ). As a result, it is

³³One can think of this force as a monopolistic underinvestment force.

³⁴If $N = 1$, there could still be overinvestment due to the moral hazard channel if $\chi > 0$.

³⁵It is a well known fact that in a standard Arrow Debreu economy with no frictions, where agents are small and take prices as given, there are no pecuniary externalities. The reason is that, in a standard Arrow Debreu economy, the change in the price is just a wealth transfer from one agent to another and, in equilibrium, the marginal utility of wealth across agents is equalized, implying that the net effect on welfare is zero. This is why the assumption that bankers are more productive than consumers (i.e. they have different marginal valuations of wealth) is crucial for the pecuniary externalities.

clear that in the case of a continuum of banks, the reason for the overinvestment is only due to the pecuniary externalities and not due to the moral hazard. The intuition is that when banks are small, they do not internalize the fact that their actions affect the bail-out they receive since the bail-out, even when targeted, depends only on the aggregate fire sale. Hence the case with a continuum of banks in this model has no moral hazard despite the presence of a targeted bail-out.³⁶

Moral Hazard

Next consider the case of $N < \infty$ and some fiscal capacity $\chi > 0$, which will be the only case where there will be moral hazard (captured by the $\frac{\partial B_{1l}^i}{\partial k_{1l}^{i,T}} = \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T}$ term in the denominator of $z_{1l}^*(k_0)$). The intuition why the moral hazard term appears in the first order condition of the banker in the decentralized equilibrium but not in the first order condition of the Central Planner is that the banker internalizes only the benefit of the bail-out while the Central Planner internalizes both the benefit and the cost of the bail-out. Since the benefit and the cost are equated in equilibrium, they cancel out from the first order conditions of the Central Planner. When banks are large and the country has some fiscal capacity, they internalize the fact that an extra dollar in the crisis state will decrease the fire sale and hence the marginal bail-out they receive. As a result, since larger investment in $t = 0$ will lead to a larger fire sale in the crisis state, in order to maximize the bail-out received, bankers choose a larger k_0 than what the Central Planner would want them to choose. The larger the moral hazard term is, which will be the case when N is small and χ is large, the larger $z_{1l}^{CP}(k_0) - z_{1l}^*(k_0)$ is, and the larger the overinvestment is due to the moral hazard (the solid red line in Figure 2 will shift down).

In general, as we increase N , it is not clear whether the overinvestment will be smaller or larger since the pecuniary externality is stronger but the moral hazard is weaker.

1.5 Decentralize the Constrained Central Planner's Allocation

In the previous section I proved that given the assumptions made, in $t = 0$, the banker overinvests relative to the constrained Central Planner and, hence, there is a role for ex-ante regulation. One way to correct for the overinvestment is to use a minimum bank capital requirement, which is the policy instrument currently used in practice by almost all countries. Banker i is required to finance at least a fraction ρ^i of his risky investment using internal equity, which, in this model, is equal to the period zero net worth of banker i ,

³⁶The linearity of the production technology of the bankers is one of the key reason why the moral hazard is not present in the case of $N \rightarrow \infty$. Assuming a concave production technology will change the result but even in that case the moral hazard will be stronger if N is smaller for the same reasons as the ones discussed in this section.

n . The minimum capital requirement constraint is given by $\rho^i k_0^i \leq n$ and will be an additional constraint that banker i will have to take into account when choosing his optimal allocation. The following proposition presents the results from the banker's optimization problem given the minimum bank capital constraint (For details see Appendix, Section A.1.2).

Proposition 1.5.1 *Given Assumptions 1.1-1.8, Assumption 1.10 and the assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, for a given exogenous minimum bank capital requirement such that $\rho > \frac{n}{k_0^*(\rho=0)}$ and considering a symmetric equilibrium, the decentralized equilibrium can be one of the following four types:*

- Type 1) $z_{1l}^*(k_{1l}^T(\rho)) = z_0^*(k_{1l}^T(\rho)) > z_{1h}^*$ if $k_{1l}^T(\rho) \in [\tilde{k}_{1l}^T, \hat{k}_{1l}^{T,\max})$*
- Type 2) $z_0^*(k_{1l}^T(\rho)) > z_{1s}^*(k_{1l}^T(\rho))$ if $k_{1l}^T(\rho) = \hat{k}_{1l}^{T,\max}$*
- Type 3) $z_{1l}^*(k_{1l}^T(\rho)) = z_0^*(k_{1l}^T(\rho)) = z_{1h}^*$ if $k_{1l}^T(\rho) = \tilde{k}_{1l}^T$*
- Type 4) $z_{1h}^* = z_0^*(k_{1l}^T(\rho)) > z_{1l}^*(k_{1l}^T(\rho))$ if $k_{1l}^T(\rho) \in [0, \tilde{k}_{1l}^T)$ where $\hat{k}_{1l}^{T,\max}$ is determined in Section A.1.4 in the Appendix. \tilde{k}_{1l}^T is unique and exists and if $0 < \tilde{k}_{1l}^T < \hat{k}_{1l}^{T,\max}$, \tilde{k}_{1l}^T is determined by $M(\tilde{k}_{1l}^T) = 0$ where*

$$M(k_{1l}^T) = \frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - 1.$$

Proof of Proposition 1.5.1. See Appendix, Section Assumption 1.9. ■

The condition that the exogenous ρ is such that $\rho > \frac{n}{k_0^*(\rho=0)}$ guarantees that the minimum capital requirement constraint will be always binding. The most interesting case to consider is to set the minimum capital requirement in such a way that in period zero the banker invests the same amount as the Central Planner would want him to invest, i.e. $k_0^* = k_0^{CP}$. Therefore, from now on let us consider the case $\rho^* = \frac{n}{k_0^{CP}}$. Notice that the minimum capital ratio is a "quantity" regulatory instrument since it directly determines the quantity of period zero investment chosen by the banker.

Given the presence of a binding ex-ante minimum bank capital requirement, Proposition 1.5.1 states that the decentralized equilibrium can be one of four types. What differentiates the equilibria is how much the banker values wealth in the high state in $t = 1$ relative to the crisis state (the low state in $t = 1$). In Proposition 1.3.2, I already proved that the only two possible borrowing contracts for the Central Planner are of Type 1 and 2. If the equilibrium is interior (of Type 1), the Central Planner always values wealth more in the crisis state than in the high state in $t = 1$ and, hence, borrows to the maximum against the high state and only then borrows against the crisis state. From Proposition 1.5.1 it is clear that a single minimum capital

requirement might not be sufficient to replicate the constrained Central Planner's allocation. In addition to choosing k_0 in $t = 0$, the banker has another degree of freedom, which is to choose how to transfer resources across states of nature and time. More precisely, he can choose how much liquidity to transfer to the crisis state, d_{1l} .³⁷ For example, if the decentralized equilibrium is of Type 4, even though $k_0^* = k_0^{CP}$, the banker optimally chooses to borrow first to the maximum against the low state and only then to borrow against the high state, which implies $d_{1l}^* > d_{1l}^{CP}$. This result is represented graphically below.

Consider parametrization where the equilibrium is of Type 1 for the Central Planner (Assumption 1.9 is satisfied) and set $\rho^* = \frac{n}{k_0^{CP}}$.³⁸

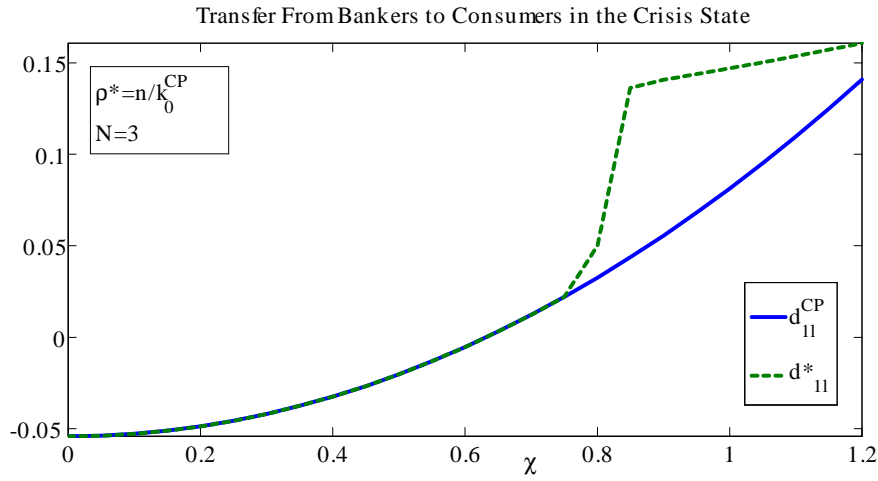


Figure 1.5

Given that $N = 3$, when the country has a large fiscal capacity, the banker optimally chooses to transfer too little liquidity into the low state relative to the Central Planner. If the dashed line in Figure 1.5 is above the solid line, the borrowing contract for the banker is either of Type 3 or of Type 4 while for the Central Planner the borrowing contract is always of Type 1. The intuition for the result is the following.

The moral hazard presents itself in two different dimensions. On the one hand, the banker is tempted to invest too much in $t = 0$ relative to the Central Planner and, on the other hand, the banker might be also tempted to pledge too high of a payment into the crisis state if the moral hazard is strong enough. The reason why this is the case is that both too much investment in period zero, k_0 , and too high of a payment

³⁷The amount of period zero borrowing is given by $\sum \pi_s d_{1s} = k_0^{CP} - n$ and also d_{2s} is determined by the borrowing constraint in $t = 2$ which is binding. Hence, what is left to decide is d_{1l} since $d_{1h} = \frac{k_0^{CP} - \pi_l d_{1l} - n}{\pi_h}$ and $k_0^* = k_0^{CP}$.

³⁸The parameters used are the same as in Figures 1.3.1 and 1.4 with the exception that $N = 3$ and I vary the fiscal capacity, χ .

pledged in the crisis state, d_{1l} , will lead to a larger fire sale which will maximize the bail-out received. This result is stated formally in the following Corollary 1.5.2.

Corollary 1.5.2 *If $N < \infty$ and $\chi > 0$, bankers realize that they affect the bail-out received both via k_0^i and d_{1l}^i , i.e. $\frac{\partial B_l^i}{\partial k_0^i} = \frac{\partial B_l^i}{\partial k_{1l}^{i,T}} \frac{\partial k_{1l}^{i,T}}{\partial k_0^i} > 0$ and $\frac{\partial B_l^i}{\partial d_{1l}^i} = \frac{\partial B_l^i}{\partial k_{1l}^{i,T}} \frac{\partial k_{1l}^{i,T}}{\partial d_{1l}^i} > 0$ where $\frac{\partial B_l^i}{\partial k_0^i}$ and $\frac{\partial B_l^i}{\partial d_{1l}^i}$ are total derivatives. Also for a given k_{1l}^T , the fewer the banks are and the larger the fiscal capacity is, the stronger the moral hazard is; $\frac{\partial^2 B_l^i}{\partial k_0^i \partial N} < 0$, $\frac{\partial^2 B_l^i}{\partial k_0^i \partial \chi} > 0$ and $\frac{\partial^2 B_l^i}{\partial d_{1l}^i \partial N} < 0$, $\frac{\partial^2 B_l^i}{\partial d_{1l}^i \partial \chi} > 0$.*

Proof of Corollary 1.5.2. See Appendix, Section *Assumption 1.9*. ■

Corollary 1.5.2 states that as long as the country has some fiscal capacity $\chi < \infty$ and the number of banks is finite, $N < \infty$, the banker internalizes the fact that his period zero actions affect the size of the bail-out, which is how the moral hazard enters the model. The moral hazard is captured by $\frac{\partial B_l^i}{\partial k_0^i} > 0$ and $\frac{\partial B_l^i}{\partial d_{1l}^i} > 0$. A large fiscal capacity and a more concentrated banking sector exacerbate the moral hazard problem. The intuition is that when the banks are large (small number of banks), they know that their marginal impact on the fire sale is large and, as a result, they affect the optimal bail-out by more, leading to a stronger moral hazard. Similarly, if a country has a larger fiscal capacity, it can afford to provide a larger bail-out, which implies that the moral hazard is stronger.

As a result, a second ex-ante instrument in the form of a limit on the payment promised in the crisis state $d_{1l}^i \leq \nu^i$, in addition to the minimum capital requirement, might be necessary to replicate the constrained Central Planner's allocation for countries with strong moral hazard. The exact conditions is specified in Proposition 1.5.3.

Proposition 1.5.3 *Consider parametrization where the equilibrium is of Type 1 for the constrained Central Planner. (i) If a) $N < \infty$ and $\chi < \chi^*(N)$ or b) if $N \rightarrow \infty$, for any χ , a minimum capital requirement (where $\rho^* = \frac{n}{k_0^{CP}}$) is sufficient to replicate the constrained Central Planner's allocation where $\chi^*(N)$ is pinned down by the system of equations $M(k_{1l}^T) = 0$ and $BC_{1l}(k_{1l}^T) = 0$ where*

$$\begin{aligned} M(k_{1l}^T) &= \frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\frac{\partial^2 \delta(B_l, \chi)}{\partial B_l \partial B_l} N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - 1 = 0 \\ BC_{1l}(k_{1l}^T) &= \pi_l [k_{1l}^T (F'(k_{1l}^T) - \theta(1 - \gamma)) + B_l(k_{1l}^T, \chi)] - \left([1 - \theta(1 - \gamma) + \pi_l(\gamma - a_{1l})] \frac{n}{\rho^*} - n \right) \end{aligned}$$

(ii) if $N < \infty$ and $\chi > \chi^*(N)$, a second instrument is required in the form of a limit on the payment pledged in the crisis state to consumers (where $\nu^* = d_{1l}^{CP}$). Part 2) (i) a) If $\chi > 0$ and $N > N^*(\chi)$ or b) if $\chi = 0$,

for any N , a minimum capital requirement (where $\rho^* = \frac{n}{k_0^{\overline{CP}}}$) is sufficient to replicate the constrained Central Planner's allocation where $N^*(\chi)$ is pinned down by the system of equations $M(k_{1l}^T) = 0$ and $BC_{1l}(k_{1l}^T) = 0$.
(ii) If $\chi > 0$ and $N < N^*(\chi)$ a second instrument is required in the form of a limit on the payment pledged in the crisis state to consumers.

Proof of Proposition 1.5.3. See Appendix, Section Assumption 1.9. ■

Proposition 1.5.3 states that a second instrument, in addition to the minimum capital requirement, is required only if the banking sector is fairly concentrated and a country has a large fiscal capacity (i.e. the second instrument is required only if $\chi > \chi^*(N)$ for a given $N < \infty$ and if $N < N^*(\chi)$ for a given $\chi > 0$). The intuition why the policy maker should optimally limit the pledged payments by the bankers in the crisis state only if the moral hazard is strong enough is the following. There are two forces which determine whether the banker values wealth more in the crisis state or in the high state in $t = 1$ and they push in different directions. The first set of forces which push towards higher valuation of wealth in the crisis state are that capital is cheaper during a crisis and also an extra dollar in the crisis state will lead to a lower fire sale, which implies higher resale value of the fire sold capital. These forces push the banker towards maximizing his net worth in the crisis state and, hence, borrowing first against the high state and only then against the low state in $t = 1$. However, the countervailing force is the benefit from maximizing the bail-out by pledging too high of a payment in the crisis state. Only once the perceived benefit of the bail-out becomes large enough (which is the case when the fiscal capacity is large and banks are large), the banker starts to value wealth less in the crisis state relative to the high state. That is why a second regulatory instrument would be required only for countries with strong moral hazard.

The way to interpret the limit on the payment pledged in the crisis state by banker i, v^i , is as a limit on the liabilities of banker i in a crisis where a bail-out is highly likely. For example, banks can sell put options which implies that they will owe payment to the buyer of those options if a stock price falls below a certain level. Also banks issue CDS contracts as well. Therefore, in order to be able to implement the regulatory instrument derived in this paper, the policy maker will need information on the payment promised by the bank in a crisis state where a bail-out will be optimally provided by the government. (For example projections from the value at risk models that banks use can be a good starting point). In other words, the limit will be on the implied payments promised via derivative contracts and other types of contracts in crisis states of nature, where an optimal bail-out will be provided ex-post.

1.6 Comparative Statics of Optimal Regulation With Respect to Fiscal Capacity

The question arises how should the optimal minimum bank capital requirement, ρ^* , and also the optimal limit on the payment pledged in the crisis state by bankers, v^* , vary across countries with different fiscal capacity. In this sub-section, I prove the key result of this paper — that smaller fiscal capacity implies a larger optimal ex-ante minimum bank capital requirement.

Proposition 1.6.1 *Conditional on Assumptions 1.1-1.8 and 1.10 and the assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, if the policy maker has an access to a sufficient number of instruments to replicate the constrained Central Planner's allocation and the parametrization is such that the Central Planner's equilibrium is of Type 1 (Assumption 1.9 is satisfied), the optimal minimum bank capital ratio is higher for more fiscally constrained countries, $\frac{\partial \rho^*}{\partial \chi} < 0$.*

Proof of Proposition 1.6.1. See Appendix, Section Assumption 1.9. ■

I already proved in Proposition 1.5.3 that the constrained Central Planner's allocation can be decentralized using a single instrument — an ex-ante minimum bank capital requirement — if the moral hazard is not too strong. A second instrument will be required — a limit on the payment promised in a crisis state by banks — if the moral hazard is strong. In this section I assume that the policy maker has a sufficient number of instruments to replicate the constrained Central Planner's allocation. The optimal minimum bank capital ratio, ρ^* , and k_0^{CP} are inversely related, $\rho^* = \frac{n}{k_0^{CP}}$. As a result, in order to prove Proposition 1.6.1, it is sufficient to prove that the Central Planner of a country with a larger fiscal capacity will optimally choose to invest more ex-ante relative to the Central Planner of a country with a smaller fiscal capacity, i.e. $\frac{\partial k_0^{CP}}{\partial \chi} > 0$. I present the intuition of the proof graphically.³⁹

³⁹The rest of the parameters are the same as in Figure 1.3.1.

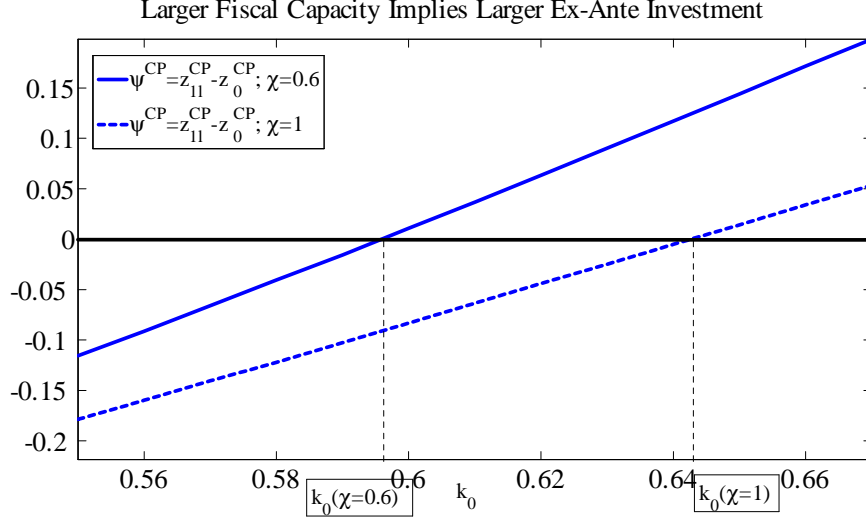


Figure 1.6

Figure 1.6 plots $\psi^{CP}(k_0, \chi)$ for two countries with different fiscal capacity. On Figure 1.6, the country with $\chi = 0.6$ has a smaller fiscal capacity and a larger marginal cost of an extra dollar of bail-out relative to the country with $\chi = 1$. Given that $\frac{\partial \psi^{CP}(k_0; \chi)}{\partial k_0} > 0$, in order to prove that the Central Planner of a country with a larger fiscal capacity would optimally choose a larger period zero investment relative to a country with a smaller fiscal capacity, it is sufficient to prove that, for a given k_0 , the dashed line is below the solid line, i.e. $\frac{\partial \psi^{CP}(\chi; k_0)}{\partial \chi} < 0$ (partial derivative). Larger fiscal capacity for a given k_0 implies a larger bail-out and a smaller fire sale during a crisis leading to a smaller inefficient transfer of resources from the bankers to the consumers. Therefore, the policy maker who can optimally afford a larger bail-out will value a dollar in the hands of the bankers in a crisis by less, $z_{1l}^{CP}(\chi = 0.6) > z_{1l}^{CP}(\chi = 1)$. Similarly, given that the policy maker of a less fiscally constrained country can contain the downside of a crisis, which would imply that the return on k_0 in a crisis is higher, from an ex-ante perspective the marginal benefit of k_0 is also higher, $z_0^{CP}(\chi = 0.6) < z_0^{CP}(\chi = 1)$. Intuitively, the policy maker in the country with the larger fiscal capacity can prop up prices by more ex-post during a crisis and control the downside, for a given level of bank assets, which implies that ex-ante he optimally chooses to have a larger investment boom ($k_0^{CP}(\chi = 1) > k_0^{CP}(\chi = 0.6)$).⁴⁰

The result that countries with larger fiscal capacity should optimally have lower ex-ante minimum bank capital requirements might appear counter-intuitive at first — large fiscal capacity implies stronger moral hazard, while the optimal policy recommendation is to regulate less ex-ante if the policy instrument is a

⁴⁰One can also calculate the optimal ρ , conditional on no minimum liquidity requirement. For countries with large fiscal capacity and a few banks (strong moral hazard), the constrained Central Planner's allocation can be no longer replicated. However, it is still the case that larger fiscal capacity implies lower ρ (or even $\rho = 0$). If the policy maker cannot prevent the banker from borrowing first against the crisis state, the optimal allocation from the problem with only an ex-ante minimum capital requirement is to borrow to the maximum, which is what the banker from the decentralized equilibrium with no ex-ante regulation would optimally choose himself.

minimum bank capital requirement. The explanation is the following. Both the pecuniary externalities and the moral hazard appear only in the first order condition of the banker with respect to k_0 in the solution of the decentralized equilibrium with no ex-ante regulation. (The banker internalizes only the benefit and not the cost of the bail-out and also the banker internalizes only partially his impact on prices.) However, the first order conditions of the banker play no role in determining the optimal minimum capital requirement or the minimum liquidity requirement. Both of these instruments are "quantity" instruments, in the sense that once the policy maker sets v and ρ , he directly determines the d_{1l} and k_0 that will emerge in equilibrium. Therefore, if the policy maker has a sufficient number of instruments to replicate the constrained Central Planner's allocation, in order to analyze the comparative statics of ρ^* (or v^*) with respect to the fiscal capacity of a country, one has to use only the first order conditions from the Central Planner's problem.⁴¹ This result will be contrasted with the result in the next section where I consider a "price" regulatory instrument instead of a "quantity" regulatory instrument.

As it turns out, *the ingredients* that generate the pecuniary externality are the key driving force behind the result that less fiscally constrained countries should have lower ex-ante minimum bank capital requirements (*but not the strength of the pecuniary externalities*). Without the assumptions that the banking sector is more productive than the consumers and that there is a fire sale (the only two assumptions necessary to generate pecuniary externalities and overinvestment), if the only source of overinvestment was the moral hazard, the optimal regulation across countries with different fiscal capacity would have been constant in this model.

Next, let us consider how v^* varies with the fiscal capacity of the country where the assumption that the policy maker has a sufficient number of instruments to replicate the constrained Central Planner's allocation is still in place.

The larger χ is, the higher the optimal amount of pledged payment in the crisis state is ($\frac{\partial v^*}{\partial \chi} > 0$). Larger fiscal capacity implies larger k_0^{CP} . Given that it is always the case that the period zero borrowing constraint binds in the high state for the Central Planner, the larger period zero investment is financed with larger ex-ante borrowing against the crisis state (or less money transferred to the crisis state). This logic explains why larger fiscal capacity implies larger v^* . The minimum liquidity requirement will be binding only for countries with fiscal capacity, χ , for which the dashed line is above the solid line in Figure 3. In that sense,

⁴¹The first order condition of the banker from the problem with no ex-ante regulation will be used only to solve for the Lagrangians of the minimum bank capital requirement constraint and the maximum amount of payment pledged in the crisis state constraint, and those Lagrangians will vary with the strength of the moral hazard and the strength of the pecuniary externality but ρ and v will not.

no regulation will be required for countries with very small fiscal capacity. However, conditional on the constraint binding, v^* increases as the fiscal capacity increases.

If one compares the US — a country with large fiscal capacity — and Switzerland — a country with small fiscal capacity — the US should optimally have a lower ex-ante minimum bank capital requirement than Switzerland which is consistent with current regulation. However, since the moral hazard might be potentially stronger in the US due to the larger fiscal capacity, US regulators might need to use a second instrument which limits the amount of CDS contracts and put options that US financial institutions can sell. If both the US and Switzerland were to need such an instrument, Swiss banks will face a tighter limit on the amount of derivative contracts that can sell.

1.7 A "Price" Instrument Versus A "Quantity" Instrument

In this section, I show that whether larger fiscal capacity implies more or less ex-ante regulation depends critically on the instrument used. I show that if the policy maker has an access to a "price" instrument such as a tax on period zero investment, the result is the opposite from the case where the instrument used is a "quantity" instrument such as a minimum bank capital requirement. If the regulatory instrument was a tax on period zero investment, larger fiscal capacity implies an optimally *higher* tax on period zero investment if the moral hazard is present.

I solve the problem of the banker using a tax on period zero investment instead of a minimum bank capital requirement. The only change in the set up is that the period zero budget constraint becomes

$$k_0^i (1 + \tau_{k_0}^i) - n + T_{k_0}^i \leq \sum_s \pi_s d_{1s}^i \quad [z_0^i] \quad (1.7.1)$$

where $\tau_{k_0}^i$ is the bank specific tax on period zero capital. The revenues from the proportional tax are distributed equally back to the bankers using the lump sum tax, $T_{k_0}^i$, which is negative and given by $T_{k_0}^i = -\sum_{i=1}^N \frac{1}{N} k_0^i \tau_{k_0}^i$. The following proposition proves that a larger fiscal capacity implies a larger tax on period zero investment as long as $1 < N < \infty$, and the tax on period zero investment is constant if $N \rightarrow \infty$ (the case with a continuum of banks and no moral hazard).

Proposition 1.7.1 *Conditional on Assumptions 1.1-1.10 and on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, if the policy maker has an access to two ex-ante instruments — an ex-ante tax on period zero investment*

("price" instrument), τ_{k_0} , and a limit on the payment promised in the crisis state, v , one can show that $\tau_{k_0} > 0$. If $N \rightarrow \infty$ (no moral hazard) then $\frac{\partial \tau_{k_0}}{\partial \chi} = 0$. If $1 < N < \infty$ then $\frac{\partial \tau_{k_0}}{\partial \chi} > 0$. τ_{k_0} and $\frac{\partial \tau_{k_0}}{\partial \chi}$ are given by

$$\tau_{k_0} = \left[\frac{z_{1l}^{CP}(k_{1l}^T)}{z_{1l}^*(k_{1l}^T, \chi)} - 1 \right] \Phi > 0 \quad (1.7.2)$$

$$\frac{\partial \tau_{k_0}}{\partial \chi} = - \frac{\partial z_{1l}^*(k_{1l}^T, \chi)}{\partial \chi} \frac{z_{1l}^{CP}(k_{1l}^T, \chi)}{[z_{1l}^*(k_{1l}^T, \chi)]^2} \Phi \geq 0 \quad (1.7.3)$$

where $\Phi = \frac{[1-\theta(1-\gamma)+\pi_l(\gamma-a_{1l})]}{(1-\frac{1}{N})} > 0$.

Proof of Proposition 1.7.1. See Appendix, Section Assumption 1.9. ■

If the instrument of choice was a tax on period zero investment, equation 1.7.2 shows that the optimal tax is positive since the banker wants to overinvest relative to the Central Planner due to both the pecuniary externality and the moral hazard. This is captured by the fact that $z_{1l}^{CP}(k_{1l}^T) > z_{1l}^*(k_{1l}^T, \chi)$ which I proved in Corollary 1.4.1.

One can show that the constrained Central Planner's allocation can be achieved using either a tax on period zero investment or a minimum bank capital requirement (conditional on imposing also a limit on the payment promised in the crisis state if necessary). However, a "price" instrument is very different from a "quantity" instrument in the way it achieves the optimal allocation. By setting τ_{k_0} , the policy maker can no longer set directly the amount of k_0^* , which was true in the case of a minimum capital requirement. τ_{k_0} affects the marginal cost of k_0 (or the "price" of k_0), as perceived by the banker, which is why a tax instrument can be thought of as a "price" instrument. In equilibrium, if the ex-ante instrument is a tax, k_0 is determined by the first order condition of the banker with respect to k_0 , which equates the marginal benefit of k_0 to the marginal cost of k_0 .

The optimal τ_{k_0} approximately equals the size of the overinvestment which is given by the scaled difference between the marginal benefit of k_0 , as perceived by the banker, minus the marginal benefit of k_0 as perceived by the constrained Central Planner. This difference is approximately equal to $\frac{z_{1l}^{CP}(k_{1l}^T) - z_{1l}^*(k_{1l}^T, \chi)}{z_{1l}^*(k_{1l}^T, \chi)}$. The larger the difference in the perceived marginal benefit of k_0 is, the larger the tax on capital has to be, in order for the policy maker to be able to replicate the constrained Central Planner's allocation. Due to the linearity assumption, the equilibrium k_{1l}^T from the Central Planner's problem does not vary with the fiscal capacity of the country (see equation 1.3.9). χ does not enter directly the marginal benefit of k_0 , as perceived by the Central Planner, since the Central Planner internalizes both the marginal cost and the marginal benefit of the bail-out and, in equilibrium, they cancel out. In contrast, χ enters directly the marginal benefit of

k_0 , as perceived by the banker. Conditional on a finite number of banks, $N < \infty$, the banker perceives the bail-out to be larger for countries with a larger fiscal capacity.⁴² Therefore, in order to achieve a given level of k_0 , the policy maker will have to increase τ_{k_0} by more for countries with larger fiscal capacity, since the perceived bail-out and, hence, the moral hazard in those countries are stronger. If one considers the case of $N \rightarrow \infty$, $\frac{\partial z_{1l}^*(k_{1l}^T, \chi)}{\partial \chi} = 0$, because there is no moral hazard. In that case, τ_{k_0} is still positive but it is no longer a function of the fiscal capacity.⁴³

In summary, the key reason why the size of the moral hazard affects the "price" instrument and not the "quantity" instrument is the following. The moral hazard enters into the model through the first order condition of the banker since the banker internalizes the benefit of the bail-out, but not the cost. If the ex-ante regulatory instrument is a tax on period zero investment, τ_{k_0} is determined by combining the first order condition of the banker with respect to k_0 , and the first order condition of the Central Planner with respect to k_0 . In contrast, when the instrument is a minimum bank capital requirement, the first order condition of the banker with respect to k_0 will no longer play a role and, hence, the strength of the externalities does not affect the optimal minimum capital requirement. (The strength of the moral hazard will affect only the Lagrangian of the minimum bank capital requirement which one can think of as a shadow tax.) This is why, as long as the policy maker can replicate the constrained Central Planner's allocation, the size of the moral hazard per se does not affect the optimal minimum capital ratio — a "quantity" instrument, but it affects the ex-ante tax on period zero investment — a "price" instrument.

1.8 Further Discussion and Conclusion

The key result derived in this paper is that cross country bank regulation should not be synchronized given the heterogeneity across countries regarding their ability to bail-out the banking sector during a financial crisis and to alleviate the costs of financial crises. This paper provides a normative result — countries with larger fiscal capacity should have lower ex-ante minimum bank capital requirements relative to countries with smaller fiscal capacity. In order to know whether one country should have lower ex-ante minimum

⁴²Mathematically, this implies that $\frac{\partial z_{1l}^*(k_{1l}^T, \chi)}{\partial \chi} > 0$ and $\frac{\partial z_{1l}^{CP}(k_{1l}^T)}{\partial \chi} = 0$.

⁴³The reason why the pecuniary externality does not affect the size of the ex-ante tax is because due to the linearity assumption the optimal $k_{1l}^{T,CP}$ does not vary with the fiscal capacity. If one were to introduce concavity in the production technology of the bankers, then there will be two opposing forces. Larger fiscal capacity will imply stronger moral hazard which will push τ_{k_0} higher. It will also imply smaller fire sale in equilibrium (unlike the linear case) and also the pecuniary externalities will be smaller which will push the ex-ante tax in the opposite direction (make it lower). In contrast, if a minimum capital requirement was used instead, the comparative static of ρ with respect to χ will be still that larger fiscal capacity implies smaller ex-ante minimum capital requirements as long as the decreasing returns to scale are not too strong during a crisis, which is a reasonable assumption.

bank capital requirement than another, a policy maker has to measure only the fiscal capacity of a country. This can be done by considering variables such as the size of the banking sector relative to the GDP, the availability of independent monetary policy and a forecast of the cost of sovereign borrowing in a crisis based on the level of sovereign debt, for example. Therefore, the result is fairly information insensitive assuming that there are no large differences across countries regarding the other parameters of the model. The second key result of the paper is that countries with large fiscal capacity and concentrated banking sectors should also impose a limit on the amount of derivative contracts that financial institutions can issue which will leave them with high liability during a systemic crisis.

One shortcoming of the model presented here is that equity is exogenous and fixed. In reality banks can adjust their equity. In a previous version of the model I explored the possibility that banks can raise costly equity and the comparative statics remain unchanged. There is a key difference between the models with and without costly equity. In the presence of costly equity, the minimum bank capital requirement is no longer sufficient to address the overinvestment since bankers end up issuing more equity than socially optimal in order to circumvent the minimum bank capital requirements. In contrast, if a "price" instrument is used, instead of a "quantity" one, the policy maker can still replicate the constrained Central Planner's allocation even in the presence of costly equity. Understanding more thoroughly the differences between the different types of instruments is left for future work.

Finally, one can make the argument that one of the main reasons why countries followed the Basel Accords and synchronized their bank regulation was to introduce a "level playing" field for their banks. This model is a legacy model and does not consider the dynamics of what might happen if I were to introduce heterogeneous regulation and banks were allowed to relocate across countries. However, usually there are large fixed costs to banks relocating either because of fixed investment in human capital and buildings or due the fact that markets are naturally segmented. The segmentation is due to the fact that monitoring costs are lower if the banks are closer to the borrowers. Given the natural market segmentation, governments have some leeway in terms of having differential minimum bank capital requirements up to a certain limit.

Chapter 2

Welfare Implications of the Structure of the Banking Sector in a Small Open Economy

2.1 Introduction

Models that explicitly introduce a banking sector, facing some type of a borrowing or a bank net worth constraint, have gained in popularity in the new macro literature (for example, Meh and Moran (2010), Gertler and Kiyotaki (2010), Gertler, Kiyotaki, and Queralto (2012), Adrian and Boyarchenko (2012), Maggiori (2013), Brunnermeier and Sannikov (2013)). At the same time, many papers have emerged that include imperfectly competitive banking sectors (for example, Gerali, Neri, Sessa, and Signoretti (2010), Hafstead and Smith (2012), Andres and Arce (2012)). After the financial crises, it has become apparent that, in order to match the empirical evidence, it is of first order importance to introduce the financial sector into standard macroeconomic models in a realistic way. It is equally important to understand what are the inefficiencies that might arise due to the specific structure of the financial sector and whether policy makers can intervene in order to improve aggregate welfare. This paper attempts to address the latter.

While economists understand well by now how the presence of future binding net worth constraints/borrowing constraints can generate pecuniary externalities and how an imperfectly competitive banking sector would lead to underinvestment, little is known how those two sources of inefficiency interact. What does this en-

vironment imply for the optimality of existing policies such as capital account controls and subsidies on the borrowing rates of firms? These are the questions that I study in this paper.

I build a finite period model of a small open economy (SOE) with a banking sector, which is monopolistically competitive and concentrated, and faces a net worth borrowing constraint. Bankers borrow from foreigners at the risk free interest rate and, in turn, lend to domestic entrepreneurs/firms using a standard debt contract (SDC). Entrepreneurs are risk neutral and are the only consumers in the economy. The implicit assumption is that domestic firms can borrow only through the domestic banking sector due to its superior monitoring technology. Entrepreneurs have an access to a concave production technology and they default on their loans with some probability, at which point the bankers seize the assets of the firm. Every period and state of nature, bankers face a net worth borrowing constraint which forces them to finance at least a fraction of the loans they provide using their own equity. There is no bank default in this economy.

First, one can compare the behavior of more and less competitive banking sectors by varying the degree of substitution of loans, for a given number of banks. There are two forces in play. On the one hand, a less competitive banking sector wants to underinvest relative to a more competitive one due to a standard monopolistic effect. However, there is also an overinvestment force which is a novel theoretical result of this paper. The overinvestment force emerges from the interaction between the desire of a less competitive banking sector to underinvest in the future and the presence of a binding net worth constraint in the crisis state in the future. The intuition is the following. Due to its desire to underinvest in the future, a less competitive banking sector does not value an extra dollar of net worth in the crisis state, when the net worth constraint binds, as much as a more competitive banking sector. As a result, it does not perceive an extra dollar invested ex-ante, which depletes the net worth of the banking sector in the crisis state, to be as costly. If this overinvestment force dominates the ex-ante standard underinvestment force, a less competitive banking sector might end up overinvesting relative to a more competitive one. This result is in contrast to the franchise value literature, which argues that it is always welfare improving to have a regulation that restricts the competitiveness of the banking sector, in addition to imposing a minimum bank capital requirement (see Keeley (1990), Hellmann, Murdock, and Stiglitz (2000) and the *Literature Review* section of this paper).

Furthermore, one can compare the decentralized allocation to the constrained Central Planner's allocation. There are two standard sources of inefficiency — pecuniary externalities and a monopolistic underinvestment force. The pecuniary externalities, which lead to overinvestment relative to the constrained Central Planner's allocation, work through two different channels. First, bankers do not fully internalize the fact that the more they lend in period zero, the lower the marginal rate of return of the other bankers is, when

the firm defaults. I call this channel a "bankruptcy" pecuniary externality and it is present even if the net worth constraint does not bind in the future. In addition, if the net worth constraint is binding in the crisis state in the future, the banker also does not internalize the fact that by lending more ex-ante, he decreases the return and the net worth of the other bankers during a crisis, which tightens the net worth constraints of the other bankers even further. I call this channel a "net worth constraint" pecuniary externality.

If the banking sector is imperfectly competitive, there is a standard monopolistic underinvestment force both in the current period and in future periods. The presence of monopolistic competition and binding net worth constraints in the crisis state in the future generate a third, novel, source of inefficiency and overinvestment in this model, which is separate from the pecuniary externalities. Under certain parametrization, even a single bank might end up overinvesting relative to the constrained Central Planner. The intuition is the following. The monopolistic bank wants to underinvest in the crisis state in the future and, as a result, does not value the marginal dollar of net worth in the crisis state as much as the Central Planner does. Therefore, the monopolistic bank is tempted to overinvest ex-ante. Whether the monopolistic bank will under-or-overinvest depends on how the standard current period underinvestment force compares to this novel overinvestment force.

Having understood the interaction between the different inefficiencies, I proceed to study how one can decentralize the constrained Central Planner's allocation. I consider two instruments that have been used by a number of emerging economies — capital account controls on inflows in the form of a tax on the borrowing interest rate from foreigners and subsidies on firm borrowing interest rates. In addition, I allow the policy maker to provide a lump sum transfer/tax to firms in order to balance his budget and I also assume commitment on behalf of the policy maker. There is an infinite number of ways to decentralize the constrained Central Planner's allocation using these instruments. Imposing the assumption that the policy maker uses only subsidies or capital account controls in every period and state of nature, I consider one way to decentralize the equilibrium. Whether the policy maker utilizes subsidies or capital account controls depends crucially on the presence of uncertainty. The overinvestment forces are potentially present only if there is future uncertainty. Therefore, there might be a role for capital account controls only for economies which can end up in a crisis state in the future with a positive probability. If there is no future uncertainty and the economy is in a steady state, only the underinvestment force is present.¹

One of the results that emerges is that there exists a country with an optimal number of banks (as a

¹I consider an implementation where the policy maker always utilizes subsidies if the banking sector is imperfectly competitive and there is no future uncertainty (even if the net worth constraint is binding for both the Central Planner and the banker in the decentralized equilibrium and no policy instrument is required).

function of the degree of loan substitution). In that economy, the overinvestment force due to pecuniary externalities completely offsets the monopolistic underinvestment force. Therefore, it is possible that no ex-ante policy intervention is required, despite the presence of ex-ante inefficiencies. However, if the pecuniary externalities are the dominant force ex-ante, a capital account tax will be required to implement the constrained Central Planner's allocation. This tax will be larger, the stronger the degree of substitution between loans is and the more banks the country has. If the underinvestment force dominates, the country should optimally impose a subsidy, which will be smaller, the larger the number of banks is and the higher the degree of substitution between loans is. Finally, it is interesting to note that even if a country has a single bank, which overinvests relative to the constrained Central Planner, due to the novel overinvestment force documented in this paper, one way to decentralize the constrained Central Planner's allocation is by using subsidies in the current period and in the future, rather than by using capital account controls.²

Literature Review

There is substantial evidence that the banking sector is imperfectly competitive (due to regulation, market segmentation, product differentiation). Claessens and Laeven (2004), Bikker and Spierdijk (2008) and Claessens (2009) estimate the degree of imperfect competition of the financial sector using Panzar and Rosse (1987)'s methodology and show that it varies significantly across countries.³ Another important parameter in the model is banking sector concentration, which could be very different from the degree of monopolistic competition of the banking sector, given that in certain cases the threat of entry is sufficient to generate a fairly competitive banking industry. In fact, the correlation between banking sector concentration and the degree of banking sector competition appears to be slightly positive in the data as shown in Figure 2.1.⁴

²It is important to note that these results represent only one way, among many, to implement the constrained Central Planner's allocation.

³The measure captures to what extent the increase in input prices affects the marginal cost and total revenue of a given bank.

⁴On the y-axis, Figure 2.1 plots the H measure of banking sector competition calculated by Claessens (2009) using Panzar and Rosse (1987)'s methodology. If the H measure is equal to 1, the financial sector is perfectly competitive and the smaller the number is, the less competitive the financial sector is. On the x-axis, Figure 2.1 plots a measure of banking sector concentration calculated by Beck, Demirguc-Kunt, and Levine (2000). The concentration measure is defined as the assets of the three largest banks as a share of assets of all commercial banks using BankScope data. (The Figure uses the numbers from the April 2013 version.)

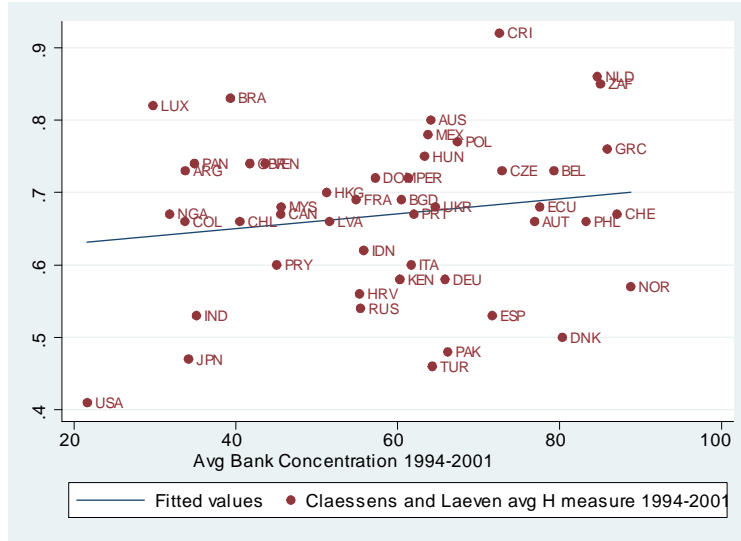


Figure 2.1

Therefore, it is important to have a model that can distinguish between the concentration of the banking sector and the degree of imperfect competition of the financial sector.

Table 2.1 shows the banking sector concentration for a number of countries, which is measured as the assets held by the largest 3 banks as a fraction of total assets in the country. While there is significant heterogeneity across countries, banking sectors appear to be very concentrated.

Table 2.1

Bank Concentration 2008

RUS	.14	IDN	.64	DNK	.79
IND	.34	IRL	.65	SVK	.8
USA	.35	COL	.65	LTU	.81
ITA	.40	ROM	.65	PER	.82
ARG	.41	MEX	.7	BHR	.82
VEN	.47	CHN	.72	QAT	.82
MYS	.49	GBR	.72	ETH	.82
THA	.5	BOL	.73	BRA	.83
LVA	.51	DEU	.74	GMB	.87
JPN	.54	POL	.75	CHE	.88
KOR	.54	BGR	.75	NER	.9
FRA	.55	NZL	.78	EST	.9
CAN	.57	ZAF	.79	HUN	.91
AUS	.61	CZE	.79	MAR	.91
AUT	.62	PHL	.8	ESP	.91

Assets of the three largest banks as a share of assets of all commercial banks.

Source: Fitch's BankScope database

Thorsten Beck and Asli Demirgüç-Kunt, 2009

The assumption that most of the foreign borrowing is channeled through the banking sector also finds support in the data. In Table 2.2, which shows bank foreign debt as a fraction of total private foreign debt

(excluding inter-firm borrowing), one can see that for many countries at least 50% of the private foreign borrowing is bank borrowing. For example, this is the case for Brazil, Hong Kong, Korea but also for many developed economies such as Greece, Austria, Germany, Sweden.

Table 2.2

Gross External Bank Debt/(Gross External Private Sector Debt Excluding Intra-firm Borrowing)					
Iceland	1	Philippines	37	Spain	64
Ecuador	2	Russia	39	Hungary	64
Argentina	10	Canada	41	EURO Area	65
Morocco	11	Czech Republic	43	Japan	65
El Salvador	12	Tunisia	44	UK	68
Mexico	13	Turkey	45	Italy	70
Moldova	17	Peru	46	Latvia	70
Indonesia	18	South Africa	46	Australia	71
India	23	Slovak Republic	47	Lithuania	72
Colombia	25	Romania	48	Portugal	73
Egypt	26	Norway	49	Germany	74
Chile	27	Israel	50	Austria	78
Kazakhstan	28	Georgia	51	Netherlands	79
Thailand	30	Malaysia	52	Finland	82
Ukraine	32	Poland	54	Denmark	83
Ireland	32	Armenia	57	Belgium	84
Bulgaria	33	Estonia	58	Sweden	85
Croatia	35	Brazil	59	Greece	87
Luxembourg	36	Slovenia	59	Jordan	88
US	36	Switzerland	64	Hong Kong	92
Belarus	37	Korea	64	Malta	93
Source: WB: Quarterly External Debt Statistics 2011Q3					

Therefore, introducing formally the financial sector is important in order to understand how foreign inflows affect domestic investment and whether there is over or underinvestment relative to the constrained Central Planner's allocation.

The types of policy instruments used by regulators to ensure financial sector stability are very different. They range from minimum bank capital requirements and capital account controls to directly regulating the competitiveness of the banking sector and subsidies for final borrowers. In this paper I focus on capital account controls in the form of a tax on foreign borrowing rates and also on subsidies on firm borrowing rates. More specifically, capital account controls that resemble a tax on foreign borrowing rates ("price" based capital account controls) have been implemented by Chile — from 1991 to 1998, Colombia — from 2007 to 2008, Thailand — from 2006 to 2008, Russia — from 2004 to 2006 and Brazil starting 2008, among others (see Ostry, Ghosh, Habermeier, Laeven, Chamon, Qureshi, and Kokenyne (2011)).⁵

Government subsidies are ubiquitous and are often distributed via subsidized bank lending rates as modelled in this paper. For example, Brazil provides subsidized interest rates for corporate loans via the

⁵The instruments used are either a direct tax or unremunerated reserve requirements (URRs), which work exactly like a tax.

Brazil's National Development Bank (BNDES). The subsidized credit amounts to about "27 percent of all productive credit". The subsidized rate at which the government lends to the BNDES is significantly lower than the nominal interest rate on government bonds. In 2008-2010, the former was 6 percent while the latter was around 12 percent. (for details and other sources see Antunes, Cavalcanti, and Villamil (2011)) The United States has the United States Small Business Administration (SBA) loan subsidy program, which guarantees loans to small businesses and provides an interest rate subsidy.⁶ For example, during the 2011 fiscal year, the subsidized new loans provided by the SBA amounted to 19.6 billion dollars (see Dilger (2011)). South Korea is another country that has relied on government subsidies on lending rates to enterprises during its industrialization period in the 70s and the 80s (see Lee (1996)).

There are a number of recent theoretical papers that explore how introducing an imperfectly competitive banking sector in standard macroeconomic models leads to a better data fit (see Gerali, Neri, Sessa, and Signoretti (2010), Hafstead and Smith (2012), Andres and Arce (2012)). There is also an older banking literature, which emphasizes imperfect banking sector competition (see Klein (1971) and Freixas and Rochet (2008) for a literature review). In the optimal bank regulation literature, Hellmann, Murdock, and Stiglitz (2000) argue that it is optimal to restrict the competitiveness of the banking sector, in addition to imposing minimum bank capital requirements. It prevents banks from overinvesting and risk shifting since they want to prevent default in order to preserve their franchise value. Keeley (1990) tests the franchise value hypothesis. The natural experiment that he examines is the financial sector liberalization in the US in the mid 1960s and he confirms the link between competitiveness and higher bank default risk. The franchise value literature encouraged policy makers of many countries to introduce reforms that decreased the degree of banking sector competition (see Boyd and DeNicolo (2005)).

However, the more recent theoretical and empirical literature disputes the franchise value policy recommendations. For example, Boyd and DeNicolo (2005) show that higher banking sector concentration could potentially lead to more default by increasing risk shifting by the final firm. Weaker financial sector competition implies higher firm borrowing rates and increased incentive of firms to maximize the upside and, hence, to risk shift. This channel was missing in Hellmann, Murdock, and Stiglitz (2000) since they considered a portfolio problem and did not model bankers and firms separately. On the empirical side, Boyd and DeNicolo (2005) show that there is no clear cut relationship between banking sector competition and financial sector stability. Using a Panzar and Rosse H-statistic measure of banking sector competition, which is a more precise measure than simply the banking sector concentration of a country, in a sample of 45 countries,

⁶"Interest rates are negotiated between the borrower and the lender but are subject to SBA maximums, which are pegged to the prime rate, the LIBOR rate, or an optional peg rate." For details see <<http://www.sba.gov/content/7a-terms-conditions>>.

Schaeck, Cihak, and Wolfe (2009) also find the opposite result — that more competitive banking sectors are less prone to systemic banking crises.

My paper contributes to the class of papers that introduces a new theoretical channel that pushes in the opposite direction of the key policy recommendation of the franchise value literature. Namely, I argue that the interaction between dynamic underinvestment and binding future net worth constraints imply that, for certain countries and states of nature, a more concentrated banking sector might end up overinvesting relative to a less concentrated one, leading to a larger loss of bank net worth in a future crisis and a more severe credit crunch.

Another strand of literature closely related to this paper are models where the financial sector faces a net worth constraint or some type of borrowing constraint. One of the key assumptions of the model developed in this paper is that the financial sector faces a net worth constraint, which the Central Planner also has to take as given. The justification why such a net worth constraint will emerge in equilibrium is similar to Holmstrom and Tirole (1997)'s "skin in the game" argument where in order to mitigate potential moral hazard, lenders require that entrepreneurs invest their own net worth. There is a growing literature which models financial institutions as facing value at risk (VaR) constraints, which are constraints internally used by financial institutions. For example, Adrian and Shin (2011) microfound the VaR constraint as a way for lenders to place a limit on the leverage of a bank in order to prevent risk shifting, which becomes equivalent to preserving a fixed probability of default. Adrian and Boyarchenko (2012) model the VaR by assuming that bankers have to retain enough equity so that a certain fraction of losses is covered. Other papers that model the financial sector as facing VaR constraints are Danielsson, Shin, and Zigrand (2011) and Brunnermeier and Pedersen (2009).

Another class of papers impose the constraint that the net present value or the net worth of the financiers has to exceed a certain value. For example, Brunnermeier and Sannikov (2013) impose the constraint that the banker's net worth cannot exceed zero. Gertler, Kiyotaki, and Queralto (2012) and Gertler and Kiyotaki (2010) rely on a moral hazard story where the bankers have an access to a less efficient technology and, as a result, in order to prevent them from using it, lenders limit the amount of loans up to the point where the net present value the bankers receive has to be greater than or equal to what they would get if they use the less productive technology. Maggiori (2013) assumes that the net present value of the bank has to be greater than or equal to zero.

Even though in this paper I assume that the financiers face a net worth constraint following the literature described above, all of the analysis could have been done with a borrowing constraint in the spirit of Kiyotaki

and Moore (1997). The contribution of my paper to both the imperfect competition literature and the bank net worth constraint/borrowing constraint literature is to combine the two and examine the welfare implications that emerge.

Finally, my paper builds on the literature that justifies the imposition of capital account controls using pecuniary externalities. International finance papers which study pecuniary externalities include Bianchi (2011), Caballero and Krishnamurthy (2001), Korinek (2010), Nikolov (2011) and Bianchi and Mendoza (2012), among others. They build on the closed economy macroeconomic models with pecuniary externalities that lead to overinvestment pioneered by Geanakoplos and Polemarchakis (1986) and Arnett, Greenwald, and Stiglitz (1994). The majority of the open economy models with pecuniary externalities do not explicitly introduce a banking sector and, as we saw in the data, a large fraction of the foreign borrowing is through the banking sector. Also in those models the borrowers are infinitesimally small while in the data the banking sectors of all countries are very concentrated. The number of banks will affect the strength of the pecuniary externality and the degree of monopolistic competition. Therefore, it is an important parameter to consider.

2.2 Model

There is a single country which is a small open economy (SOE). There are three periods, $t = 0, 1, 2$ and there is uncertainty only in the middle period, $t = 1$. In $t = 1$, the economy can be in the high state (a boom) with a probability π_H and in the low state (a recession) with a probability $\pi_L = 1 - \pi_H$. The shock is an aggregate shock. There are four types of agents — entrepreneurs/borrowers, bankers, foreign lenders and a policy maker. There are two goods — a capital good and a consumption good where the price of consumption is the numeraire good and is set equal to one.

There is a continuum of risk neutral entrepreneurs distributed uniformly on $[0, 1]$. They borrow from the banks using a short term standard debt contract (SDC). In some states of nature in $t = 1$, the entrepreneurs can default upon which their assets will be seized by the bankers. The entrepreneurs are the equity owners of the banks, but I assume that they cannot borrow from the banks they own equity in.⁷ Entrepreneurs have limited liability. If they default, they will be still allowed to borrow again and produce (no exclusion

⁷One can also think of this set up as having a representative family that splits into two agents in the beginning of period $t = 0$ (a banker and an entrepreneur who runs the firm) and they get back together at the end of period $t = 2$ and consume (see Gertler and Kiyotaki (2010)). Alternatively, I could have modelled bankers and entrepreneurs as agents that consume independently and the Central Planner places an exogenous weight on each agent. The intuition behind all the results will remain unchanged.

from debt markets for any amount of time).⁸ Entrepreneurs invest in $t = 0, 1$ and produce with a lag. The loans are imperfect substitutes and entrepreneurs can transform the consumption good into the capital good one-for-one. For simplicity, I assume that the entrepreneurs consume all the profits every period after producing and that the capital good depreciates one hundred percent after producing. These assumptions are not crucial for the final results but allow me to derive analytical results.

The banking sector is monopolistically competitive where the number of banks is finite and equal to n . In the environment that I consider there will be no default by the banking sector. Banks maximize the dividends they pay to entrepreneurs. Bankers also face a net worth constraint, which limits the amount they can borrow from foreigners. I also assume that there is an infinite number of risk neutral foreign lenders that can lend money only to the domestic banks and cannot directly lend to entrepreneurs.⁹ In expectation, foreign lenders receive the world risk free interest rate, R_t^f .

2.2.1 The Problem of the Representative Entrepreneur

First, let us consider the problem of the entrepreneur. Every period the entrepreneur consumes, borrows from bankers and invests where the contract between the banker and the entrepreneur is a standard debt contract (SDC). In period t , the banker offers a lending rate, \bar{R}_t^l , and the entrepreneur takes it as given and chooses how much to borrow, L_t . The representative entrepreneur maximizes his utility given by

$$\max_{C_t, K_t, L_{i,t}} E_0 \sum_{t=0}^2 C_t \quad (2.2.1)$$

subject to the period t budget constraint

$$A_t K_{t-1}^\alpha - R_t^l L_{t-1} + L_t + D_t \geq C_t + K_t \quad (2.2.2)$$

where the discount rate between periods is equal to one and C_t is the consumption of the entrepreneur. The entrepreneur has an access to a decreasing returns to scale production technology where period t output is given by $A_t K_{t-1}^\alpha$. $A_1 = A_L$ if the low state is realized and $A_1 = A_H$ if the high state is realized where $A_H > A_L$. $A_2 = \bar{A}$ and is the same in both states of nature. K_t is period t investment which produces in $t + 1$, D_t are the dividends paid by the banks which the entrepreneur takes as given. L_t is the amount of

⁸This assumption is not crucial and one can re-write the problem by modelling the entrepreneurs as overlapping generations instead or one can interpret the set up as if there is an entry of new entrepreneurs in case of aggregate default.

⁹One can endogenize this assumption by assuming that domestic banks have a better information regarding domestic investment projects relative to foreign investors or they have a better monitoring technology, for example.

new aggregate loans, $R_t^l L_{t-1}$ is the payment made on loans taken in $t-1$. The ex-post return on period $t-1$ loans is equal to $R_t^l = \min \left\{ \frac{A_t K_{t-1}^\alpha}{\bar{L}_{t-1}}, \bar{R}_{t-1} \right\}$ where $\hat{L}_{t-1} = \sum_{i=1}^n \frac{1}{n} L_{i,t-1}$ is a simple average. $R_t^l = \frac{A_t K_{t-1}^\alpha}{\bar{L}_{t-1}}$ is the ex-post return if the firm defaults¹⁰ and $R_t^l = \bar{R}_{t-1}$ is the ex-post return if the firm does not default. I assume that in every period, the profits of the firm (if any) and also the dividends received from the banks are consumed, $C_t = A_t K_{t-1}^\alpha - R_t^l L_{t-1} + D_t$, which implies $L_t = K_t$. This assumption is made only as a simplification in order to derive intuitive analytical results.¹¹ Assume that loans from different banks are imperfect substitutes which allows us to model the banking sector as monopolistically competitive. The CES aggregator over loans is given by

$$L_t = \left[\sum_{i=1}^n \frac{1}{n} (L_{i,t})^{\frac{(\rho-1)}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad (2.2.3)$$

where $L_{i,t}$ is the amount of loans the entrepreneur takes from bank i and $\rho \in (1, \infty)$ is the elasticity of substitution between loans. If $\rho \rightarrow \infty$, all the loans are perfect substitutes and if $\rho \rightarrow 1$ the functional form approaches Cobb Douglas which has an elasticity of substitution of one. The total expenditure on loans for each entrepreneur, if there is no default, is given by

$$\bar{R}_t^l L_t = \sum_{i=1}^n \frac{1}{n} \bar{R}_{i,t}^l L_{i,t} \quad (2.2.4)$$

The assumption that the entrepreneur consumes all the profits every period after producing and that capital depreciates one hundred percent makes the problem of the entrepreneur static. However, since I assume that in $t = 1$ there is no longer uncertainty and in $t = 0$ there is uncertainty I will write the $t = 0$ and the $t = 1$ problems separately to define notation and highlight the difference in the "uncertainty" versus the "no uncertainty" case.

I solve the model backwards. In $t = 1$, the uncertainty is resolved and since there will be no default in $t = 1$, then $R_2^l = \bar{R}_1$ and the entrepreneur maximizes

$$\max_{L_{i,1}} \left[\bar{A} (L_1(s_1))^\alpha - \bar{R}_1^l(s_1) L_1(s_1) + D_2(s_1) \right] + A_1 L_0^\alpha - R_1^l(s_1) L_0 + D_1(s_1)$$

subject to equations 2.2.3 and 2.2.4. All the variables are a function of the period one state, s_1 , which can

¹⁰The implicit assumption is that if the firm defaults on one bank, it defaults on all of them and that all banks receive a share of the output equal to the share of their loans out of total loans (since all the loans are equally productive).

Note that I assume that if the firm defaults, the bankers can seize only the output of the firms and not the dividends paid. (However, this assumption is irrelevant since dividends will be optimally paid only in the last period and, at that point, there will be no default since all the uncertainty is resolved in the middle period.)

¹¹Of course, the larger the net worth of the entrepreneur is, which he can reinvest, the lower the welfare loss is due to the fact that the banking sector is imperfectly competitive. However, as long as there is some borrowing in equilibrium, the qualitative results presented here remain.

be either high or low. Also \bar{A} is the period two TFP shock which is known in $t = 1$ and for simplicity \bar{A} is the same after either state of nature. One can think of \bar{A} as a steady state productivity. Section B.1.1 in the Appendix presents the details of the optimization problem. The first order condition with respect to $L_{i,1}$ gives the standard demand for loans equations. The demand for aggregate loans is determined by banks equating the marginal cost of loans to the marginal benefit of loans (which here is simply the marginal benefit of capital since $K_t = L_t$).

$$\alpha \bar{A} (L_1)^{\alpha-1} = \bar{R}_1^l \quad (2.2.5)$$

The first order condition with respect to $L_{i,1}$ can be re-written as

$$L_{i,1} = L_1 \left[\frac{\bar{R}_{i,1}^l}{\bar{R}_1^l} \right]^{-\rho} \quad (2.2.6)$$

which determines the demand for bank specific loans and is standard in models with monopolistic competition. The interest rate on aggregate loans is given by

$$\bar{R}_1^l = \left[\sum_{i=1}^n \frac{1}{n} (\bar{R}_{i,1}^l)^{(1-\rho)} \right]^{\frac{1}{(1-\rho)}} \quad (2.2.7)$$

Given that the problem is static, the period zero problem of the entrepreneur is given by

$$\max_{L_{i,0}} \pi_H [A_H (L_0)^\alpha - \bar{R}_0^l L_0] + \pi_H D_1 (s_H) + \pi_L D_1 (s_L) + D_0$$

where I assume for simplicity that the entrepreneur has no net worth in period zero and $K_{-1} = 0$. The model will be parametrized in such a way that there is always firm default in the low state. As a result, due to limited liability, entrepreneurs maximize only their profits in the non-default state since they lose their output in the case of default. Entrepreneurs will take period zero and period one dividend payments as given, which, in equilibrium, will be equal to zero. The first order conditions are very similar to the ones in period $t = 1$, with the only difference being that the aggregate demand for loans is given by $\alpha A_H (L_0)^{\alpha-1} = \bar{R}_0^l$. The equations for $L_{i,0}$ and \bar{R}_0^l are identical but for the time subscript.

What is important to note is that the demand schedule for loans will be downward sloping and the imperfectly competitive banking sector will internalize that. This result, combined with the fact that if there is no default by the firm bankers can seize only a fraction of the output of the firm, will generate the standard underinvestment channel due to imperfect competition.¹²

¹²It will be important that the fraction of the output that the banker will seize in equilibrium in the high state in $t = 1$ is exactly equal to the marginal product of capital.

2.2.2 Banker i 's Optimization Problem

In this section I solve the optimization problem of banker i . There are n banks and they provide loans that are imperfect substitutes. The banking sector is monopolistically competitive. It is important that this model allows us to separate the bank concentration effect, proxied by n , from the imperfect competition effect, proxied by ρ .

I solve the problem of banker i backwards. In period $t = 1$, banker i maximizes his net worth in $t = 2$, which is also equal to the dividends paid to the entrepreneurs, $N_{i,2} = D_{i,2}$.¹³ Since all the uncertainty is resolved in $t = 1$, there will be no default in $t = 2$. Banker i takes the first order conditions of the entrepreneur and the actions of the other bankers, $L_{j,1}$, as given, where $j \neq i$. Given $N_{i,1}$, the optimization problem of banker i in $t = 1$ becomes

$$\max_{L_{i,1}} \bar{R}_{i,1}^l (L_{i,1}) L_{i,1} - R_1^f [L_{i,1} - N_{i,1}]$$

subject to a net worth constrain which states that at least a fraction η of the loans that bank i issues have to be financed using the bank's own net worth

$$N_{i,1} \geq \eta L_{i,1} \quad [\lambda_{i,1}]. \quad (2.2.8)$$

$\lambda_{i,1}$ is the Lagrangian of the borrowing constraint and it represents the marginal value of an extra dollar of net worth in period one as perceived by the banker. Aghion, Banerjee, and Piketty (1999) provide one way to microfound the functional form of the net worth constraint that I specify in equation 2.2.8. Instead of a net worth constraint, one could use a borrowing constraint as an alternative friction, which is used by the literature on pecuniary externalities. The borrowing constraint can be expressed using a similar functional form as equation 2.2.8 with the main difference being that η would be endogenous and time varying. The rest of the analysis would be similar.

Notice that banker i takes into account that his actions affect the return that he receives, $\bar{R}_{i,1}^l (L_{i,1})$. The state variable is the period one net worth, $N_{i,1}$, which is a function of the realized state. If the firms default in $t = 1$, the period one net worth of banker i is given by

$$N_{i,1}(s_L) = A_L(L_0)^\alpha \frac{L_{i,0}}{\hat{L}_0} - R_0^f [L_{i,0} - N_{i,0}]$$

¹³Banker i will optimally choose not to pay dividends prior to $t = 2$ since entrepreneurs are assumed to always consume them, which implies that the marginal value of dividends is one. The marginal value of the bank's net worth will be always greater than or equal to one. Notice that all the results will go through if bankers and entrepreneurs are treated as separate agents and then the assumption that dividends have to be consumed when paid out can be relaxed.

and if the firms do not default, the period one net worth of banker i is

$$N_{i,1}(s_H) = \bar{R}_{i,0}L_{i,0} - R_0^f [L_{i,0} - N_{i,0}]$$

If the net worth constraint does not bind ($\lambda_1 = 0$), and after imposing a symmetric equilibrium, the first order condition with respect to $L_{i,1}$ becomes

$$\bar{R}_1^l = \underbrace{\gamma}_{\text{mark up}} \underbrace{R_1^f}_{MC}$$

where $R_1^f \geq 1$ is the period one risk free world interest rate which is also the marginal cost (MC) of an extra dollar of loans provided by banker i . The mark-up is constant and is given by the following equation

$$\gamma = \frac{1}{\left(1 - \frac{1}{\rho} \left(1 - \frac{1}{n}\right) - (1 - \alpha) \frac{1}{n}\right)} \geq 1 \quad (2.2.9)$$

The key result to notice is that if the net worth constraint is not binding, the first order condition is not a function of period zero variables and

$$L_1 = \left[\frac{\alpha \bar{A}}{\gamma R_1^f} \right]^{\frac{1}{1-\alpha}}$$

Let us consider the mark-up. Since $\left(1 - \frac{1}{\rho}\right)n > \left(1 - \alpha - \frac{1}{\rho}\right)$, it will be the case that $0 < \frac{1}{\gamma} \leq 1$. The mark-up is a function of the number of banks, n , the degree of substitution between loans, ρ , and the productivity of capital captured by α . Notice that the larger ρ is (the more competitive the banking sector is), the lower the mark up is $\frac{\partial \gamma}{\partial \rho} < 0$. Whether the mark-up decreases or increases as n increases depends on whether $1 - \alpha > \frac{1}{\rho}$. If $1 - \alpha > \frac{1}{\rho}$, more banks implies lower mark-up $\frac{\partial \gamma}{\partial n} < 0$. Finally, the higher α is, the smaller the mark up is, $\frac{\partial \gamma}{\partial \alpha} < 0$. The latter result is intuitive since the larger α is, the larger the fraction of output that accrues to the banker is (which is clear from equation 2.2.5), and the smaller the incentive to underinvest. If $\alpha \rightarrow 1$ and if $n = 1$ (the banker fully internalizes his effect on output), then $\gamma = 1$. If both $\rho \rightarrow \infty$ and $n \rightarrow \infty$, the banking sector is perfectly competitive, in which case the equilibrium interest rate charged converges to the world risk free rate $\bar{R}_1^l = R_1^f$ and $\gamma = 1$. If $n \rightarrow \infty$, the mark up converges to the standard monopolistic competition mark-up with a continuum of banks given by $\gamma = \frac{\rho}{\rho-1}$. If $n = 1$, $\gamma = \frac{1}{\alpha}$ which coincides with the monopolistic case.

If the net worth constraint binds ($\lambda_1 > 0$), and after imposing a symmetric equilibrium, the amount of period one loans is determined by the net worth constraint

$$L_1 = \frac{1}{\eta} N_1$$

where

$$\lambda_1 = \frac{1}{\eta} \left[\frac{1}{\gamma} \bar{R}_1^l - R_1^f \right]$$

and $\bar{R}_1^l = \alpha \bar{A} (L_1)^{\alpha-1}$. The marginal value of an extra dollar of bank net worth, λ_1 , as perceived by the banker, is larger, the smaller the mark-up, γ , is. The banker will value his net worth more, the more competitive the economy is (the smaller the mark up is). The marginal value of an extra dollar of bank net worth will be important when I discuss how the decentralized equilibrium compares to the constrained Central Planner's allocation and also how the competitive decentralized equilibrium compares to the non-competitive one. In order to make the problem interesting, throughout the rest of the paper, I assume that the period one net worth constraint never binds in the high state in $t = 1$.

Next I solve for the optimal lending of banker i at $t = 0$. Banker i takes as given his optimal actions in both states of nature in $t = 1$. He also takes the actions of the other bankers in the economy as given. Banker i maximizes his expected dividend payment in the last period

$$\begin{aligned} \max_{L_{i,0}} E_0 N_{i,2} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}] &= \max_{L_{i,0}} E_0 \left[\bar{R}_{i,1}^l (L_{i,0}) - R_1^f \right] L_{i,1} (L_{i,0}) \\ + R_1^f \left(\pi_H \bar{R}_{i,0}^l (L_{i,0}) + (1 - \pi_H) A_L L_0^\alpha \frac{1}{\hat{L}_0} - R_0^f \right) L_{i,0} &+ R_1^f R_0^f N_{i,0} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}] \end{aligned}$$

where $\bar{R}_{i,1}^l (L_{i,0})$ implies that $\bar{R}_{i,1}^l$ is a function of $L_{i,0}$ and so are $L_{i,1} (L_{i,0})$ and $\bar{R}_{i,0}^l (L_{i,0})$. One way to interpret the objective function of the bank in $t = 0$ is the following. The net worth in $t = 2$ is the sum of the expected profits in periods $t = 1$ and $t = 2$ plus the return on the starting net worth of banker i . Consider parametrization where the net worth constraint does not bind in $t = 1$ in the high state, $\lambda_1 (s_H) = 0$.¹⁴ I proved that $L_{i,0}$ does not affect the optimal $L_{i,1}$ if the net worth constraint is not binding in $t = 1$, which will be the case in the high state in $t = 1$. I also assume that the economy starts in normal times where the net worth constraint does not bind in $t = 0$, $\lambda_0 = 0$. Assuming no default by banker i in the high state, after imposing a symmetric equilibrium, the first order condition with respect to $L_{i,0}$ can be summarized as (for details see Appendix, Section B.1.2)

¹⁴The most interesting case will be when the constraint binds in the crisis state $\lambda_1 (s_L) > 0$ as well, which will generate a recession and a credit crunch, but for now the equations presented are more general.

$$MC(L_0) = -(1 - \pi_H) \lambda_1(s_L) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = R_1^f E_0 \frac{\partial N_{i,1}(s_1)}{\partial L_{i,0}} = MB(L_0) \quad (2.2.10)$$

where

$$\frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = A_L L_0^{\alpha-1} \left(1 - \frac{1}{n} (1 - \alpha) \right) - R_0^f < 0 \quad (2.2.11)$$

$$\frac{\partial N_{i,1}(s_H)}{\partial L_{i,0}} = \alpha A_H L_0^{\alpha-1} \frac{1}{\gamma} - R_0^f > 0 \quad (2.2.12)$$

and if $\lambda_1(s_L) > 0$

$$\lambda_1(s_L) = \frac{1}{\eta} \frac{\partial N_{i,2}(s_L)}{\partial L_{i,1}(s_L)} = \frac{1}{\eta} \left[\alpha \bar{A}(L_1(s_L))^{\alpha-1} \frac{1}{\gamma} - R_1^f \right] > 0 \quad (2.2.13)$$

L_0 is determined by equating the expected marginal profit of an extra dollar of L_0 , $MB(L_0)$ ¹⁵, to the marginal cost of an extra dollar of L_0 , $MC(L_0)$. If the banker lends one extra dollar in $t = 0$ to the entrepreneur, the marginal benefit is the increase of expected period one profits, $R_1^f E_0 \frac{\partial N_{i,1}(s_1)}{\partial L_{i,0}}$. However, there is a cost associated with lending more in $t = 0$ if the net worth constraint binds in the low state in $t = 1$. An extra dollar lent in $t = 0$ implies that in the low state, when the entrepreneur defaults, the net worth of the bank will decrease by $-\frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}}$. If the net worth constraint binds in the low state in $t = 1$, the marginal value of an extra dollar of net worth in the low state is measured by the Lagrangian of the borrowing constraint in the low state $\lambda_1(s_L)$. Hence, the marginal cost associated with lending one more dollar in $t = 0$ is $-(1 - \pi_H) \lambda_1(s_L) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}}$ which is the cost of making the borrowing constraint tighter. Notice that depending on the degree of monopolistic competition, ρ , and the degree of banking sector concentration, n , the equilibrium L_0 will be different. Denote the decentralized equilibrium with a star.

Definition 2.2.1 *The non-competitive decentralized symmetric equilibrium is defined as a vector of quantities $\{C_1^*(s_1), C_2^*(s_1), K_0^*, K_1^*(s_1), L_0^*, L_1^*(s_1)\}_{s_1 \in \{s_L, s_H\}}$ and prices $\{\bar{R}_0^l, \bar{R}_1^l(s_1)\}_{s_1 \in \{s_L, s_H\}}$ such that*

- *The markets for loans, capital and the consumption good clear*
- *Banker i chooses $L_{i,t}$ to maximize his expected net worth in $t = 2$ taking into account the demand schedules for loans of the entrepreneurs, the net worth constraint and taking as given the actions of the other bankers $L_{j,t}$ where $j \neq i$*

¹⁵Note that the marginal benefit is net of the interest rate payment to foreigners.

- The representative entrepreneur chooses $\{C_1^*(s_1), C_2^*(s_1), K_0^*, K_1^*(s_1), L_0^*, L_1^*(s_1)\}_{s_1 \in \{s_L, s_H\}}$, in order to maximize the profits of the firm taking as given the loan interest rates $\{\bar{R}_0^l, \bar{R}_1^l(s_1)\}_{s_1 \in \{s_L, s_H\}}$
- Foreign investors inelastically supply risk free loans to the domestic banking sector at the exogenous world interest rate R_t^f

Next, I prove existence and uniqueness and compare how the optimal investment varies with the degree of banking sector competition.

Proposition 2.2.2 (i) If the parametrization is such that there is a crisis in the low state in $t = 1$ and no crisis in the high state in $t = 1$ and in $t = 0$, $\lambda_1(s_L) > 0, \lambda_1(s_H) = 0, \lambda_0 = 0$, the equilibrium is unique and exists. (ii) Countries with a more competitive banking sector will borrow and invest more than countries with a less competitive banking sector, $\frac{\partial L_0^*}{\partial \rho} > 0$, if

$$\underbrace{-(1 - \pi_H) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} \frac{\partial \lambda_{i,1}(s_L)}{\partial \rho}}_{= \frac{MC(\rho; L_0^*)}{\partial \rho} > 0} < \underbrace{R_1^f \pi_H \frac{\partial^2 N_{i,1}(s_H)}{\partial L_{i,0} \partial \rho}}_{= \frac{MB(\rho; L_0^*)}{\partial \rho} > 0} \quad (\text{Assumption 2.1})$$

and $\frac{\partial L_0^*}{\partial \rho} < 0$ if Assumption 2.1 is not satisfied.

Proof of Proposition 2.2.2: See Appendix, Section B.1.5.

What is surprising about the result in Proposition 2.2.2 is that if Assumption 2.1 is not satisfied, countries with more competitive banking sectors (higher ρ) borrow and invest less (not more) than countries with less competitive banking sectors. The reason why one would expect less competitive banking sectors to invest less is that lower competition (smaller ρ) strengthens the standard underinvestment channel.

A less competitive banking sector would optimally want to underinvest in the current period ($t = 0$) and the future period ($t = 1$) relative to a more competitive banking sector. The current period underinvestment leads to the expected perceived period one profits to be lower for less competitive banking sectors, which is captured by the term $\frac{\partial MB(\rho; L_0^*)}{\partial \rho} > 0$. However, the combination of future desire to underinvest and a binding net worth constraint in the crisis state in the future leads to an overinvestment force in $t = 0$ relative to the more competitive case. The intuition is the following. A less competitive banking sectors wants to also underinvest in the future, in period one. As a result, an extra dollar in the crisis state in the future becomes less valuable for the less competitive banking sector, given that the perceived marginal value of an extra dollar of net worth is lower ($\frac{\partial \lambda_{i,1}(s_L)}{\partial \rho} > 0$). Therefore, the perceived marginal cost of $L_{i,0}$ is actually smaller,

the less competitive the banking sector is, where an extra $L_{i,0}$ is costly because it depletes the net worth of the bank in the crisis state $\left(\frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} < 0\right)$. This latter overinvestment channel is captured by the term $\frac{MC(\rho; L_0^*)}{\partial \rho} > 0$.

In summary, there are two forces in play when comparing a less competitive banking sector to a more competitive banking sector. The first force is a standard current period ($t = 0$) underinvestment force. The second one is an overinvestment force due to the combination of the future underinvestment force (period $t = 1$) and a binding net worth constraint in the crisis state in the future. Therefore, the answer to the question whether a more or a less competitive banking sector experiences a larger investment boom ex-ante (prior to a crisis) will depend crucially on the interaction of the two effects. This result is in contrast to the franchise value literature, which argues that less competitive banking sectors are less prone to overinvestment since banks are concerned about preserving their franchise value. While the model in this paper endogenously produces no bank default, the overinvestment force that I identify here (which is present even in the case of a single bank) will be a countervailing force to the franchise value argument.

2.2.3 Constrained Central Planner's Problem

In order to determine the source of the inefficiencies in this economy, in this section I solve the constrained Central Planner's (CP's) problem. The CP faces the same constraints as the banker in the decentralized equilibrium. He has to take into account the same net worth constraints that the bankers face and also the first order conditions of the entrepreneurs.¹⁶ The CP chooses the amount of loans provided by every banker, taking into account that the equilibrium played is symmetric. There are two sources of inefficiency. The first source is pecuniary externalities, which will lead to overinvestment. The strength of the pecuniary externalities will vary with the number of banks, n . The second inefficiency is due to monopolistic competition, which will lead to underinvestment. The strength of the underinvestment will vary with the degree of loan substitution, ρ , and the number of banks, n .

Solving the problem of the CP backwards, in $t = 1$, the CP maximizes the total welfare of the entrepreneurs, who also own the banks, subject to the banker's net worth constraint. The problem simplifies to maximizing total output since the entrepreneurs, who are the only agents consuming in the economy, are risk neutral. (See Appendix, Section B.1.3 for detailed derivations of the CP's problem.)

¹⁶In this specific model, it does not make a difference whether the CP takes the first order conditions of the entrepreneurs as an additional constraint or not. However, this assumption is important for other models with a third agent, such as a consumer, in order to prevent transfer of resources from consumers to constrained bankers.

$$\max_{L_1} \bar{A}L_1^\alpha - R_1^f [L_1 - N_1] + \lambda_1^{CP} [N_1 - \eta L_1]$$

The first order condition with respect to L_1 is given by

$$\alpha \bar{A}L_1^{\alpha-1} - R_1^f - \lambda_1^{CP} \eta = 0$$

If the net worth constraint does not bind ($\lambda_1^{CP} = 0$), the entrepreneur will invest up to the point where the marginal cost of an extra dollar of loans equals the marginal benefit of an extra dollar of loans from the Central Planner's point of view, $\alpha \bar{A}L_1^{\alpha-1} = R_1^f = \bar{R}_1^f$. If the constraint binds ($\lambda_1^{CP} > 0$), then

$$L_1 = \frac{1}{\eta} N_1$$

where $N_1(s_L) = A_L L_0^\alpha - R_0^f (L_0 - N_0)$ and $N_1(s_H) = \alpha A_H L_0^\alpha - R_0^f (L_0 - N_0)$. The marginal value of an extra dollar of net worth when the constraint is binding in period one is given by the Lagrangian $\lambda_1^{CP} = \frac{1}{\eta} (\alpha \bar{A}L_1^{\alpha-1} - R_1^f)$. The optimization problem in $t = 0$ is to maximize the sum of expected period one and period two output.

$$\begin{aligned} \max_{L_0} & (1 - \pi_H) \left(\bar{A} (L_1(s_L))^\alpha - R_1^f L_1(s_L) \right) + \pi_H \left(\bar{A} (L_1(s_H))^\alpha - R_1^f L_1(s_H) \right) \\ & + R_1^f \left(E_0 A_1 L_0^\alpha - R_0^f L_0 \right) + R_1^f R_0^f N_0 + \lambda_0^{CP} [N_0 - \eta L_0] \end{aligned}$$

Considering only parametrization where $\lambda_0^{CP} = 0$, $\lambda_1^{CP}(s_L) \geq 0$ and $\lambda_1^{CP}(s_H) = 0$, the first order condition with respect to L_0 , is given by¹⁷

$$MC^{CP}(L_0) = -(1 - \pi_H) \lambda_1^{CP}(s_L) \frac{\partial N_1(s_L)}{\partial L_0} = R_1^f \left(E_0 A_1 \alpha (L_0)^{\alpha-1} - R_0^f \right) = MB^{CP}(L_0) \quad (2.2.14)$$

where $\frac{\partial N_1(s_L)}{\partial L_0} = (\alpha A_L (L_0)^{\alpha-1} - R_0^f) < 0$. If the bank net worth constraint in period one is binding, $\lambda_1^{CP}(s_L) > 0$,

$$\lambda_1^{CP}(s_L) = \left[\alpha \bar{A} (L_1(s_L))^{\alpha-1} - R_1^f \right] \frac{1}{\eta} > 0$$

The way to interpret the first order condition is the following. An extra dollar invested in period zero

¹⁷In the Appendix, Section B.1.3 I prove existence and uniqueness.

will increase expected period one output, net of foreign debt payment, by MB^{CP} . However, there is an extra cost associated with an extra dollar invested in $t = 0$. In the low state in $t = 1$, an extra dollar invested in $t = 0$ will decrease the net worth of the banker by $-\frac{\partial N_1(s_L)}{\partial L_0}$ and the value of an extra dollar of net worth in the low state as perceived by the CP is given by $\lambda_1^{CP}(s_L)$. Throughout the rest of the paper, the superscript CP will denote the optimal allocation of the Central Planner.

2.3 Central Planner's Allocation vs Decentralized Equilibrium

In this section I compare the CP's allocation against the decentralized equilibrium. I show that there are two sources of inefficiency in this environment — pecuniary externalities, which lead to overinvestment relative to the constrained CP's allocation, and imperfect competition of the banking sector, which leads to underinvestment relative to the constrained CP's allocation.

To preview the results in this section, there will be two different channels through which the pecuniary externalities will generate overinvestment in this model. The intuition behind the first channel is that each banker does not fully internalize the fact that the more he invests in period zero, the more he decreases the marginal return of all other bankers when the representative entrepreneur defaults in period one. I will call this type of pecuniary externalities *"bankruptcy"* pecuniary externality. It will lead to overinvestment even if there is no binding net worth constraint in the future. In addition, if the banker's net worth constraint is binding in the crisis state in the future, the pecuniary externalities will lead to overinvestment through a second channel. The more each banker invests in period zero, the lower the return and the net worth of all other bankers is in the crisis state, which will tighten the net worth constraints of all other bankers. I will call this second type of pecuniary externalities *"net worth constraint"* pecuniary externality. Since the Central Planner maximizes total output, he internalizes both of those externalities. The reason why I refer to these externalities as pecuniary externalities, is because one can think of the return each banker receives if the entrepreneur defaults as a price which, in equilibrium, is equal to the output of the entrepreneur divided by total loans. Since this price depends on the total output of the firm and the firm borrows from many different banks in order to invest (an assumption implicit in the monopolistic competition environment), the banker realizes that his actions only partially affect this price. Each banker perceives his effect on the rate of return during a crisis to be larger, the smaller n is. Therefore, a more concentrated banking sector implies weaker pecuniary externalities.

The second source of inefficiency in this model is due to the imperfect competition of the banking sector combined with the fact that the banker appropriates only part of the firm's output. The banker maximizes

only his own net worth/dividend payments and does not internalize the fact that the more he lends to the firm, the higher the profits of the entrepreneur are in the state of nature where the entrepreneur does not default.

Finally, I show that even if there are no pecuniary externalities, $n = 1$ (or they are weaker — n is small), a monopolistic bank might still overinvest relative to the CP as a result of the interaction between the desire to underinvest in the future in the crisis state and a binding net worth constraint during a crisis. The intuition why this is the case is the following. Given that the banker wants to underinvests in the future, he does not value an extra dollar of net worth in the crisis state as much as the CP does. Therefore, an extra dollar of loans provided in period zero, which decreases the net worth of the banker in a crisis, is perceived to be less costly. Whether the monopolistic bank ends up lending too much relative to the constrained CP depends on how this overinvestment force compares to the classic underinvestment force in period zero.

2.3.1 Bank Net Worth Constraint Never Binds

First, I examine the case where the net worth constraint does not bind either for the CP or the banker in the decentralized equilibrium in any period or state of nature.

Proposition 2.3.1 *If the net worth constraint does not bind for $\forall t$ and in any state of nature for either the CP or the banker in the decentralized equilibrium, the decentralized equilibrium exhibits underinvestment relative to the constrained CP's allocation, $L_0^{CP} > L_0^*$, if*

$$\pi_L A_L \left(\frac{1}{n} [\alpha - 1] + 1 \right) + \pi_H \alpha A_H \frac{1}{\gamma} < \alpha E_0 A_1 \quad (\text{Assumption 2.2})$$

and overinvestment, $L_0^{CP} < L_0^$, if Assumption 2.2 is violated. In the monopolistic case ($n = 1$), $L_0^{CP} > L_0^*$ and in the perfectly competitive case, ($n \rightarrow \infty$ and $\rho \rightarrow \infty$), $L_0^{CP} < L_0^{CE}$, where CE stands for competitive equilibrium.*

Proof of Proposition 2.3.1. One can simplify the first order conditions of the CP and the banker when the net worth constraint never binds and write them as

$$MB(L_0) = R_1^f \left(\pi_L A_L L_0^{\alpha-1} \left(\frac{1}{n} [\alpha - 1] + 1 \right) + \pi_H \alpha A_H L_0^{\alpha-1} \frac{1}{\gamma} - R_0^f \right) = 0 \quad (2.3.1)$$

$$MB^{CP}(L_0) = R_1^f \left(E_0 A_1 \alpha (L_0)^{\alpha-1} - R_0^f \right) = 0$$

where $\gamma = \frac{1}{(1-\frac{1}{\rho})(1-\frac{1}{n})-(1-\alpha)\frac{1}{n}}$. Since both $MB'(L_0) < 0$ and $MB^{CP'}(L_0) < 0$ (For proof see Appendix, Proof of Proposition 2.2.2 and Section B.1.3), there will be underinvestment if $MB^{CP}(L_0) > MB(L_0)$, which will be true if Assumption 2.2 is satisfied. If $n = 1$, then $\gamma = \frac{1}{\alpha}$ and Assumption 2.2 will be always satisfied. If $n \rightarrow \infty$ and $\rho \rightarrow \infty$, Assumption 2.2 is violated. ■

If $n = 1$, the banker will underinvest relative to the CP as a result of a standard underinvestment channel where in the high state only a fraction of the output accrues to the entrepreneur. Since, the banker optimizes only his own profits and does not internalize the fact that higher investment increases the profits of the entrepreneur as well, he will underinvest relative to the constrained CP, who maximizes total output. It is surprising that in the perfectly competitive case, $n \rightarrow \infty$ and $\rho \rightarrow \infty$, the banker overinvests relative to the Central Planner even when the net worth constraint is not binding in any t and state of nature. The intuition for this result is the following. When the entrepreneur defaults in the low state, bankers appropriate all of the output. In the perfectly competitive case, every banker takes the return received in the crisis state as given.¹⁸ As a result, every banker does not internalize the fact that the more he lends, the more he decreases the marginal rate of return of the other bankers (due to the concavity of the production technology of the firm) when the entrepreneur defaults.¹⁹ This is why this model will exhibit pecuniary externalities which lead to overinvestment even if the net worth constraint does not bind. I call this type of pecuniary externalities "*bankruptcy*" pecuniary externality.

For any other combination of n and ρ , whether there is over-or-underinvestment depends on whether Assumption 2.2 is satisfied, which determines whether the underinvestment force dominates the overinvestment force. Notice that the less competitive the banking sector is (small ρ), the more likely it is that Assumption 2.2 is satisfied since $\gamma'(\rho) < 0$ and hence the more likely it is that the economy exhibits underinvestment.

2.3.2 Binding Bank Net Worth Constraint In a Crisis

In this subsection I compare the decentralized equilibrium and the constrained Central Planner's allocation assuming that the net worth constraint binds in the low state in $t = 1$ but does not bind in the high state

¹⁸In terms of the actual math, due to the assumption about constant elasticity of substitution (CES) of loans, which is required in order to model monopolistic competition, every entrepreneur borrows from all bankers in order to invest. Therefore, the return in the crisis state is a function of aggregate variables which the bankers are too small to affect.

¹⁹In the perfectly competitive case, the banker also does not internalize the fact that he affects the return received by the other bankers in the high state where there is no default. However, in the high state, in equilibrium, the return the banker receives equals the marginal product of capital (and not the whole output as in the default state). Therefore, the overinvestment force is exactly offset by the underinvestment force due to the fact that the banker maximizes only a fraction of output in the high state while the CP's objective function is to maximize the whole output in the high state. The intuition is similar as to why in a standard model with no default where the banker receives the marginal product of capital in every state of nature the CP's allocation coincides with the CE.

in $t = 1$ and in $t = 0$. Essentially, this case maps to starting the economy in normal times where with some probability next period there will be a credit crunch crisis due to a binding bank net worth constraint, which would be the most interesting case to consider.

Perfectly Competitive Case, $n \rightarrow \infty, \rho \rightarrow \infty$

In the perfectly competitive case, given the assumptions made above, I show that bankers always overinvest relative to the CP due to two sources of pecuniary externalities. The first one was described in Section 2.3.1 and is present even if the net worth constraint in the low state is not binding. The second source of overinvestment occurs due to the binding net worth constraint in the crisis state.

Proposition 2.3.2 *If $n \rightarrow \infty, \rho \rightarrow \infty$ and the net worth constraint binds only in the low state in $t = 1$ for both the CP and the banks in the decentralized equilibrium, then $L_0^{CP} < L_0^{CE}$.*

Proof of Proposition 2.3.2. When $n \rightarrow \infty$ and $\rho \rightarrow \infty$, the only difference between the first order conditions of the CP and the banker in the competitive equilibrium is that $\left(\frac{\partial N_1(s_L)}{\partial L_0}\right)^{CP} < \left(\frac{\partial N_1(s_L)}{\partial L_0}\right)^{CE}$. This inequality implies that $MB^{CP}(L_0) < MB^{CE}(L_0)$ and $MC^{CP}(L_0) > MC^{CE}(L_0)$ which combined with $MC'(L_0) > 0$ and $MB'(L_0) < 0$ is sufficient to prove that for any parametrization that leads to the borrowing constraint binding for both the CP and the banker in the low state in $t = 1$, the banker will end up overinvesting relative to the CP.²⁰ ■

On the one hand, the banker overinvests because he does not internalizes the fact that by investing too much in period zero he will decrease the marginal return the other bankers receive when the entrepreneur defaults (captured by $MB^{CP}(L_0) < MB^{CE}(L_0)$). This is the same source of overinvestment as in the case where the net worth constraint does not bind and is what I refer to as "*bankruptcy*" pecuniary externality.

The intuition behind the second source of the pecuniary externality is due to the binding net worth constraint in the crisis state and is captured by $MC^{CP}(L_0) > MC^{CE}(L_0)$. Every banker does not internalize the fact that the more he invests in period zero, the lower the return of all other bankers is when the firm defaults. This leads to lower net worth and tighter net worth constraints of the other bankers. The CP internalizes this externality. I refer to this second type of pecuniary externality as "*net worth constraint*" pecuniary externalities.

²⁰Note that there will be still overinvestment even if the constraint does not bind in the future for the CP but binds in the crisis state for the banker from the CE.

Monopolistic Bank, $n = 1$

It is informative to consider the monopolistic case, since $n = 1$ implies that the pecuniary externalities are turned off — the banker internalizes fully his impact on the return of the loans. The standard monopolistic force pushes towards underinvestment in both $t = 0$ and $t = 1$. However, the binding net worth constraint in the crisis state introduces a dynamic aspect to the problem of the monopolist. The interaction between the binding net worth constraint in the crisis state (low state in $t = 1$) and the underinvestment force in the crisis state can lead to overinvestment in $t = 0$ relative to the CP's allocation even when $n = 1$. The result is formally stated in Proposition 2.3.3 below.

Proposition 2.3.3 *If $n = 1$ and the net worth constraint binds only in the low state in $t = 1$ for both the CP and the monopolistic bank, then $L_0^{CP} > L_0^*(n = 1)$ if*

$$\underbrace{R_1^f (1 - \alpha) \pi_H A_H \alpha L_0^{*\alpha-1}}_{>0} + \underbrace{(1 - \pi_H) \frac{1}{\eta} \left(\alpha A_L (L_0^*)^{\alpha-1} - R_0^f \right) [1 - \alpha] \bar{A} (L_1^*(s_L))^{\alpha-1}}_{<0} > 0 \quad (\text{Assumption 2.3})$$

where $L_1^* = \frac{1}{\eta} \left(A_L (L_0^*)^\alpha - R_1^f [L_0 - N_0] \right)$. If Assumption 2.3 is violated, the monopolist overinvests relative to the Central Planner $L_0^{CP} < L_0^*(n = 1)$.

Proof of Proposition 2.3.3. See Appendix, Section B.1.5. ■

The intuition why it is possible for the monopolist to overinvest relative to the CP is very similar as to why the monopolist might end up investing more in $t = 0$ than a perfectly competitive banking sector, which I discussed in Section 2.2.2. On the one hand, there is the standard underinvestment force which pushes the monopolist to want to underinvest relative to the CP in both periods zero and one. However, when the period one underinvestment force is combined with a binding net worth constraint in the crisis state, an overinvestment force emerges. The banker, unlike the CP, does not value an extra dollar of net worth in the crisis state as much as the CP since he wants to underinvest relative to the CP in the crisis state. Hence, he perceives the marginal cost of an extra dollar of L_0 to be smaller than the CP does, $MC(L_0(n = 1)) < MC^{CP}(L_0)$, which is why there is an overinvestment force. Whether the monopolist ends up overinvesting or not depends on the relative strength of the two forces. For example, as $\pi_H \rightarrow 1$, then the overinvestment force vanishes since the crisis state disappears and Assumption 2.3 is always satisfied. In contrast as $\eta \rightarrow 0$, which captures the degree of financial development (since lower η implies that bankers can finance a larger fraction of loans using foreign loans rather than internal equity), the overinvestment force dominates. Therefore, in the limit, as it becomes very easy to finance domestic lending using bank

debt, the overinvestment force dominates. Notice that these are only limiting results since L^* is a function of η and π_H as well.

General Case

Having gained a better understanding of the externalities present in this model and whether they push towards over-or-underinvestment relative to the constrained CP's allocation, next let's consider the general case for any n and ρ . In the general case, whether $L_0^{CP} < L_0^*$ or not will be determined by the relative strength of the overinvestment and the underinvestment forces. The overinvestment forces are due to the pecuniary externalities – both the "bankruptcy" and the "net worth constraint" pecuniary externality — and the combination of future desire to underinvest in the crisis state plus a binding net worth constraint in the crisis state. The underinvestment force is the standard imperfect competition underinvestment force in period zero.

Proposition 2.3.4 *For any n and ρ and if the net worth constraint binds only in the low state in $t = 1$ for both the CP and banker in the decentralized equilibrium, then $L_0^{CP} > L_0^*$ if*

$$\begin{aligned}
& \underbrace{R_1^f \pi_H A_H \alpha \left(1 - \frac{1}{\gamma}\right) L_0^{*\alpha-1}}_{\text{underinvestment in } t=0 \geq 0} - \underbrace{R_1^f (1 - \pi_H) A_L (1 - \alpha) \left(1 - \frac{1}{n}\right) L_0^{*\alpha-1}}_{\text{"bankruptcy" pecuniary externality} \leq 0} + \quad (\text{Assumption 2.4}) \\
& \quad - \underbrace{(1 - \pi_H) \frac{1}{\eta} \left[\alpha \bar{A} (L_1^* (s_L))^{\alpha-1} - R_1^f \right] (1 - \alpha) \left(1 - \frac{1}{n}\right) A_L L_0^{*\alpha-1}}_{\text{"net worth constraint" pecuniary externality} \leq 0} \\
& \quad - \underbrace{(1 - \pi_H) \frac{1}{\eta} \left[R_0^f - \left(1 - (1 - \alpha) \frac{1}{n}\right) A_L L_0^{*\alpha-1} \right] \alpha \bar{A} (L_1^* (s_L))^{\alpha-1} \left(1 - \frac{1}{\gamma}\right)}_{\text{underinvestment in } t=1 \leq 0} > 0 \quad (2.3.2)
\end{aligned}$$

where $L_1^* = \frac{1}{\eta} \left(A_L (L_0^*)^\alpha - R_1^f [L_0^* - N_0] \right)$. If Assumption 2.4 is violated, then the banker overinvests relative to the Central Planner $L_0^{CP} < L_0^*$.

Proof of Proposition 2.3.4. See Appendix, Section B.1.5. ■

If Assumption 2.4 is satisfied then the banker in the decentralized equilibrium underinvests relative to the CP, $L_0^{CP} > L_0^*$, and if it's not, he overinvests, $L_0^{CP} < L_0^*$. The monopolistic competition pushes towards underinvestment in period $t = 0$. However, while the period zero underinvestment force makes it more likely for the banker to want to underinvest, the period one underinvestment force actually pushes towards overinvestment as is apparent in the formula for Assumption 2.4 presented above. The intuition is similar to the $n = 1$ case — the fact that bankers want to underinvest in $t = 1$ in the crisis state relative to the CP

implies that they value an extra dollar of net worth by less than the CP. Hence an extra dollar of bank loans provided in period zero, which leads to lower net worth in the crisis state, is less costly from the perspective of the banker. From the formula for Assumption 2.4 one can also see the two pecuniary externalities forces both of which push towards overinvestment.

2.4 Decentralize the Constrained Central Planner's Allocation

In this section I consider how the CP's allocation can be decentralized. There are many different ways to do that. Two policy instruments that are often used in practice are loan subsidies to firms/entrepreneurs and also capital account controls which limit the inflow of funds into the country. The latter can be implemented using a tax on the return paid on foreign loans, which is the approach that I follow in this paper. I will assume that the only instruments available to the policy maker are subsidies on entrepreneur's borrowing rates, $\tau_{i,t}^s$, a tax on the interest rate paid to foreigners, $\tau_{i,t}^{cc}$ and lump sum transfers (taxes if negative), $T_{i,t}$, to the bankers, where i stands for banker i . To be more precise, when the firm is borrowing from bank i , the effective interest rate it faces is given by $(1 - \tau_{i,t}^s) \bar{R}_{i,t}^l$ and when banker i borrows from foreigners, the effective rate of return he has to pay is $(1 + \tau_{i,t}^{cc}) R_t^f$. I assume commitment, which implies that all instruments, $\{\tau_{i,t}^s, \tau_{i,t}^{cc}, T_{i,t+1}\}_{t=0,1}$, are determined in the beginning of period $t = 0$, before entrepreneurs and bankers make any decisions. Hence, they take these instruments as given. The government will balance its budget every period, which will be achieved via the lump sum transfers to the bankers. In period $t + 1$, if the entrepreneur does not default, he will receive from the government the effective subsidy on the loans he took in period t . Similarly, the government will receive the tax from previous period loans of the bankers from the foreigners. The lump sum transfer to banker i in $t + 1$, if there is no default by the representative entrepreneur, is given by $T_{i,t+1} = \tau_{i,t}^{cc} R_t^f [L_{i,t} - N_{i,t}] - \tau_{i,t}^s \bar{R}_{i,t}^l L_{i,t}$ where $L_{i,t} - N_{i,t}$ is the amount borrowed by banker i from foreigners. If the entrepreneur defaults, then the banker seizes all the output of the firm and the firm cannot even pay the subsidized interest rate payments. In that case $T_{i,t+1} = \tau_{i,t}^{cc} R_t^f [L_{i,t} - N_{i,t}]$. The subsidy has to be paid only conditional on the loan being repaid and, hence, will be paid only in the high state.²¹ For detailed derivation of the Ramsey Problem, see Section B.1.4 in the Appendix.

In this model, τ_t^{cc} and τ_t^s are not uniquely pinned down, which is intuitive since the banker will either underinvest or overinvest relative to the CP. As a result, only one of those instruments in every period and state of nature will be sufficient to replicate the constrained CP's allocation. However, for every t and state of nature, there are many possible combinations of $\tau_t^{cc} > 0$ and $\tau_t^s > 0$ which implement the CP's allocation.

²¹The exact specification is not crucial for the results.

If one observes both subsidies and capital account controls in practice, it does not mean that the policies are necessarily sub-optimal.

One can make the argument that there are political and monetary costs to providing subsidies and collecting taxes. In this paper, I will not solve for the policy problem that would minimize those costs, but will consider one possible implementation, instead, that provides interesting intuition and meets the following criteria. First, at every point in time and in every state of nature, the policy maker can use only one instrument — either a subsidy or capital account controls. Also I assume that if there is no uncertainty in the future and the banking sector is imperfectly competitive, the policy maker always uses a subsidy. I would like to emphasize that this is only one way to implement the constrained CP's allocation. Given the assumptions made, Proposition 2.4.1 specifies the optimal policy.

Proposition 2.4.1 *Assuming the policy maker can commit and the net worth constraint binds in the crisis state, the CP's allocation can be decentralized using a lump sum transfer to entrepreneurs, T_t , subsidy on entrepreneurs' borrowing rates, $\tau_t^s \geq 0$, and a capital account control in the form of a tax on banker's borrowing rates from foreigners, $\tau_t^{cc} \geq 0$. One possible implementation of the constrained Central Planner's allocation is given by: $\tau_1^s = 1 - \frac{1}{\gamma}$, $\tau_1^{cc} = 0$. If $\tilde{\tau}_0^{cc}(\tau_0^s = 0) > 0$, then $\tau_0^{cc} = \tilde{\tau}_0^{cc}(\tau_0^s)$ and $\tau_0^s = 0$. If $\tilde{\tau}_0^{cc}(\tau_0^s = 0) < 0$, then $\tau_0^{cc} = 0$ and $\tau_0^s > 0$, where τ_0^s is pinned down by $\tilde{\tau}_0^{cc}(\tau_0^s) = 0$ and*

$$\begin{aligned} \tilde{\tau}_0^{cc}(\tau_0^s) = & \underbrace{-\Phi R_1^f \pi_H \alpha A_H \left(1 - \frac{1}{\gamma(1 - \tau_0^s)}\right)}_{\text{"period 0 underinvestment"}} + \underbrace{\Phi R_1^f (1 - \pi_H) A_L (1 - \alpha) \left(1 - \frac{1}{n}\right)}_{\text{"bankruptcy" pecuniary externality} \geq 0} \\ & + \underbrace{\Phi (1 - \pi_H) \left[\alpha \bar{A} (L_1^{CP}(s_L))^{\alpha-1} - R_1^f\right] \frac{1}{\eta} A_L (1 - \alpha) \left(1 - \frac{1}{n}\right)}_{\text{"net worth constraint" pecuniary externality} \geq 0} \end{aligned}$$

$\Phi = \frac{1}{(L_0^{CP})^{1-\alpha} R_0^f [R_1^f + (1-\pi_H) [\alpha \bar{A} (L_1^{CP}(s_L))^{\alpha-1} - R_1^f] \frac{1}{\eta}]} > 0$ and CP stands for the optimal allocation of the CP.

Proof of 2.4.1. See Appendix, Section B.1.5. ■

In order to understand the optimal policy, it is worth discussing how to interpret the three period model, $t = 0, 1, 2$, where all the uncertainty is resolved in $t = 1$. As a reminder, the structure of the model allows me to study the following sequence of events:

1) In period $t = 0$, agents face an uncertain period one output. For example, the presence of uncertainty can be justified by the discovery of a new technology in $t = 0$ the returns of which are unknown.

2) In $t = 1$, the uncertainty is realized and the economy can end up in a crisis state with probability $1 - \pi_H$ if the technology was not very productive or in a non-crisis state with probability π_H . If in the crisis

state, the entrepreneur defaults and the bankers' net worth is depleted to the point where their net worth constraints are binding. This environment proxies an economy with high aggregate default rates and a credit crunch.

3) In $t = 2$, the economy converges to a steady state where there is no future uncertainty regarding the productivity of the technology and the probability of default in $t = 2$ goes to zero.²² This latter assumption is fairly realistic when we think about the aggregate economy in normal times.

Let's analyze the optimal policy backwards. In $t = 1$ given that there is no future uncertainty and no future default or binding net worth constraints, all overinvestment forces are shut down. The only externality is the underinvestment force which is present only if the banking sector is imperfectly competitive and the optimal subsidy in period one is equal to $\tau_1^s = 1 - \frac{1}{\gamma}$. If the banking sector is perfectly competitive, then no intervention is required since $\gamma = 1$.²³

The interesting question is what should be the optimal policy when there is a significant uncertainty regarding the productivity of the new technology and with some probability it can turn out to be unproductive, leading to large aggregate default and binding net worth constraints in the future. This is the environment that the policy maker and the agents face in $t = 0$. In $t = 0$, there will be overinvestment forces due to the pecuniary externalities and an underinvestment force if the banking sector is imperfectly competitive. (Notice that the third source of overinvestment will be no longer present in $t = 0$ since it was a result of the interaction between the future binding net worth constraint and the desire of bankers to underinvest in $t = 1$. However, by imposing a subsidy in the crisis state equal to $\tau_1^s = 1 - \frac{1}{\gamma}$, the period one underinvestment force was shut down.) Whether the policy maker will use capital account controls or subsidies in period zero depends on whether the underinvestment or the overinvestment force dominates which will be determined by whether $\tilde{\tau}_0^{cc}(\tau_0^s)$ is positive or negative.

First, notice that it is also possible that no regulation is required in $t = 0$ if the overinvestment and underinvestment forces exactly offset each other and $\tilde{\tau}_0^{cc}(\tau_0^s = 0) = 0$. This result is formally stated in the following Corollary 1.

Corollary 2.4.2 *For every country, where the net worth constraint binds in the crisis state for both the CP and the banker in the decentralized equilibrium, there exists $n^*(\rho)$, such that no period zero regulation is*

²²Also the definition of a steady state would imply no binding net worth constraints in $t > 2$ if more periods were to be included.

²³Note that if the net worth constraint is binding for both the CP and the banker in $t = 1$ in the crisis state no policy intervention would be required. This would be an alternative way to decentralize the constrained CP's allocation. Introducing a subsidy in $t = 1$ in the low state if the banking sector is imperfectly competitive, even if not required, will change the period zero problem in an interesting way, which is why I choose to take this approach.

necessary, $\tau_0^{cc} = 0$ and $\tau_0^s = 0$.

The n^* , which one can solve for using the equation $\tilde{\tau}_0^{cc}(\tau_0^s = 0) = 0$, is given by

$$n^*(\rho) = 1 + \frac{R_1^f \pi_H \alpha A_H (1 - \alpha)}{\left(R_1^f + \left[\alpha \bar{A} (L_1^{CP}(s_L))^{\alpha-1} - R_1^f \right] \frac{1}{\eta} \right) (1 - \pi_H) A_L (1 - \alpha) - R_1^f \pi_H \alpha A_H \frac{1}{\rho}}$$

where $L_1^{CP} = \frac{1}{\eta} \left(A_L (L_0^{CP})^\alpha - R_1^f [L_0^{CP} - N_0] \right)$. L_0^{CP} and L_1^{CP} are not a function of either n or ρ since the structure of the banking sector does not affect the CP's problem. Also notice that the degree of monopolistic competition, $\gamma = \frac{1}{(1-\frac{1}{\rho})(1-\frac{1}{n})-(1-\alpha)\frac{1}{n}}$, is governed by two key parameters – ρ and n . Therefore, even if the loans are perfect substitutes, $\rho \rightarrow \infty$, as long as $n < \infty$, the banking sector is still imperfectly competitive — $\gamma > 1$.²⁴ The larger the degree of substitution between loans, the smaller is the number of banks for which no period zero regulation will be required, $n^*(\rho) < 0$. The intuition for this result is that larger ρ weakens the underinvestment force while smaller n strengthens the underinvestment force and weakens the pecuniary externality.

From equation A.1.36, one can see that the smaller n is, the more likely it is that in period zero the optimal policy is to have subsidy since smaller n makes the underinvestment force stronger and also weakens the pecuniary externalities. Similarly, the smaller ρ is, the stronger the underinvestment force is and the more likely it is that a subsidy will be required. In addition to comparative statics with respect to ρ and n , one can consider the limiting cases of $\alpha \rightarrow 1$ and $\eta \rightarrow 0$. $\alpha \rightarrow 1$ affects the concavity of the production technology, which in turn affects the pecuniary externalities of this problem. The pecuniary externalities work through the fact that a single banker, if small, does not internalize the fact that the more he invests in L_0 , the lower the marginal rate of return of the other bankers is in the crisis state. This leads to lower direct returns of other bankers – “*bankruptcy*” *pecuniary* externality – and if the net worth constraint binds in the crisis state, to tighter net worth constraints for the other bankers – “*net worth constraint*” *pecuniary* externality. If the production technology approaches a linear production technology, $\alpha \rightarrow 1$, then a single banker's actions no longer affect the marginal return of the other bankers and the pecuniary externalities will disappear, i.e. $\tilde{\tau}_0^{cc} = 0$. If $\eta \rightarrow 0$, it is easier for bankers to lever and the “*net worth constraint*” *pecuniary* externality is stronger. As a result, $\tilde{\tau}_0^{cc} \rightarrow \infty$.

Next I provide the explicit formulas for subsidies and capital account taxes. Conditional on correcting for future underinvestment using a subsidy in $t = 1$, if a country has only a few banks and/or a small degree of

²⁴If $\rho \rightarrow \infty$, then this environment maps to a model where the banking sector faces Cournot competition.

substitution between loans, in $t = 0$, the policy maker should use a subsidy and not capital account controls. The formula for the optimal period zero subsidy conditional on $\tilde{\tau}_0^{cc}(\tau_0^s) < 0$ is given by

$$\tau_0^s = 1 - \frac{\frac{1}{\gamma} R_1^f \pi_H \alpha A_H}{R_1^f \pi_H \alpha A_H - (1 - \pi_H) A_L (1 - \alpha) \left(1 - \frac{1}{n}\right) \left[R_1^f + \left[\alpha \bar{A} (L_1^{CP}(s_L))^{\alpha-1} - R_1^f \right] \frac{1}{\eta} \right]} \quad (2.4.1)$$

One can show that $\tau_0^{sf}(\rho) < 0$ and $\tau_0^{sf}(n) < 0$. Also if $n = 1$, then the optimal policy in period zero is a subsidy given by $\tau_0^s = 1 - \alpha$ and also a subsidy in $t = 1$, $\tau_1^s = 1 - \alpha$. What is interesting to note about this result is that even if the monopolist wants to overinvest, due to the interaction between future underinvestment and binding future net worth constraints, one set of optimal instruments to correct for this overinvestment are actually subsidies in period $t = 0$ and in period $t = 1$ rather than capital account controls.²⁵

In contrast, if the banking sector has a lot of banks and loans are highly substitutable, then capital account controls would be required in period zero conditional on optimal subsidies being implemented in $t = 1$ and the optimal capital account tax is given by

$$\tau_0^{cc} = -\Phi R_1^f \pi_H \alpha A_H \left(1 - \frac{1}{\gamma}\right) + \left(R_1^f + \left[\alpha \bar{A} (L_1^{CP}(s_L))^{\alpha-1} - R_1^f \right] \frac{1}{\eta} \right) \Phi (1 - \pi_H) A_L (1 - \alpha) \left(1 - \frac{1}{n}\right)$$

Conditional on a capital account controls being required, the tax is larger, the higher the degree of substitution between loans is $\tau_0^{cc}(\rho) > 0$ and the larger number of banks is $\tau_0^{cc}(n) > 0$.

2.5 Further Discussion

In practice, policy makers have an access to another instrument, in addition to the capital account controls, that can help them control overinvestment – minimum bank capital requirement. Minimum bank capital requirements require that banks finance at least a fraction of their loans using equity. Therefore, this policy instrument appears to be similar to the bank net worth constraint in this model, which both the bankers and the CP take as exogenous and given. However, there is one crucial difference. The bank net worth constraint is imposed exogenously by foreign lenders to prevent the banker and the CP from diverting part

²⁵Notice that if $n = 1$ and there was ex-ante overinvestment and the policy maker had chosen to use no subsidy in $t = 1$ in the crisis state since the net worth constraint is binding, the period zero optimal policy would have been capital account controls. This would be an alternative way to implement the allocation to the case I examine in this paper.

or all of the borrowed amount to be used for a project that the foreigner cannot seize, for example (see the discussion on net worth constraints in the literature review).²⁶ In contrast, the minimum bank capital requirement is determined by the policy maker (CP) in order to correct for overinvestment. Therefore, those two constraints are very different.

One might ask the question why not simply use minimum bank capital requirements to correct for the overinvestment instead of imposing capital account controls. In the model I develop in this paper, equity is exogenous and fixed. However, if I were to allow bankers to raise costly equity, the minimum bank capital requirement will be no longer sufficient to replicate the constrained Central Planner's allocation. Bankers will be tempted to raise too much costly equity in order to circumvent the minimum bank capital requirement and lend more than the socially optimal amount. In contrast, the capital account controls if implemented as a tax on foreign borrowing rates, as specified in this paper, will be sufficient to replicate the constrained CP's allocation even if costly equity is introduced in the model. Therefore, it is not surprising that even though emerging markets impose minimum bank capital requirements, they also rely on capital account controls.

²⁶I don't model explicitly the moral hazard due to the desire of the banker (and also the CP) to divert resources but the literature on net worth constraints has given different examples of how this can be done.

Chapter 3

The Continuing Puzzle of Exchange Rate Forecasting (Joint with Kenneth Rogoff)

3.1 Introduction

Understanding the connection between exchange rates and macroeconomic fundamentals has been one of the central challenges in international macroeconomics since the start of the modern floating exchange rate era in the early 1970s. Although exchange rates are indeed an asset price, and, therefore, highly volatile, they also reflect basic macroeconomic fundamentals such as interest rates, purchasing power, and trade balances. As such, international economists have long held out hope they could explain exchange rates better than, say, finance economists can explain the absolute level of stock prices. If so, the results would be of enormous help to policy-makers including, for example, central bankers who might worry about the effect of monetary policy on exchange rates.

Unfortunately, in practice, the performance of structural exchange rate models has been frustratingly disappointing. As first shown by Meese and Rogoff (1983a), models that perform well in-sample seldom do so out-of-sample. Although one can find some forecasting power at horizons of two to four years (e.g., Meese and Rogoff (1983b), Mark (1995) or Engel, Mark, and West (2007)), attempts to forecast at more policy-relevant horizons of one month to one year have been far less successful.¹

¹Further research is required to determine the robustness of the long-horizon forecastability results with respect to using

Indeed, until recently, there had been surprisingly little progress despite hundreds of studies using a plethora of techniques (see Cheung, Chinn, and Pascual (2005) for a survey). Lately, however, the literature has experienced a new life. A growing number of papers have been reporting somewhat more positive short-term forecasting results by implementing panel forecast methods, innovative estimation procedures, more powerful out-of-sample test statistics and new structural models. These include influential papers by Gourinchas and Rey (2007), Engel, Mark, and West (2007) and Molodtsova and Papell (2009) along with many other notable studies.² This paper re-examines the new evidence and considers a number of variations and refinements.

We conclude that despite notable methodological improvements, the euphoria has been exaggerated by *misinterpretation of some newer out-of-sample test statistics for nested models, over-reliance on asymptotic out-of-sample test statistics and failure to check for robustness to the time period sampled.*

Our examination of the most popular exchange rate forecasting structural models and specifications leads us to conclude that one of the sources of the overly optimistic results is the failure to check robustness with respect to alternative out-of-sample test statistics. In the presence of forecast bias³, the new tests for nested models⁴ cannot be always interpreted as minimum mean square forecast error tests.⁵ As we show, in certain cases, this is a first-order problem. Furthermore, while new asymptotic out-of-sample tests such as the Clark-West are attractive due to their simplicity, bootstrapped out-of-sample tests remain more powerful and better sized.⁶ Finally, even if the results remain statistically significant if one considers alternative out-of-sample test statistics, all of the structural models and specifications we review fail to produce robust forecasts over different sample periods, implying that in one period the random walk is a better forecaster and in another the structural model outperforms the random walk. In such cases, even if a structural model performs well during the most recent period of time, there is no guarantee that the relationship will be

different sub-samples. For instance, Mark (1995)'s results do not hold when one updates his sample (see Kilian (1999)).

²See also Rapach and Wohar (2002), Rossi (2006), Groen (2005), Groen (2007), Cerra and Saxena (2010), Ardic, Ergin, and Senol (2008), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2010) and Sellin (2007).

³In this paper we are concerned only with "scale" bias as opposed to "location" bias. In other words, our result refers only to the cases where the forecast systematically over or under-predicts the observed value by a certain percent (see Holden and Peel (1989) for a distinction between the two types of bias). For a general definition of forecast bias see Marcellino (2000), pp. 534.

⁴These tests include the tests developed by Clark and West (2006), Clark and West (2007), Clark and McCracken (2001) and Clark and McCracken (2005).

⁵This is a problem with both the asymptotic and the bootstrapped Clark-West and Clark-McCracken

⁶The advance of the literature on time series bootstrapping and the increase of computational power have made the bootstrap an increasingly attractive alternative to asymptotic inference (see Berkowitz and Kilian (2000), Kilian (1999), Mark and Sul (2001), MacKinnon (2002), Brownstone and Valletta (2001) and Politis and White (2004)). For a detailed discussion of how the bootstrap can provide a significant improvement over asymptotic inference see Li and Maddala (1997).

preserved in the future.

The paper is organized in the following way. Section 3.2 sets out our criteria for what constitutes a "good" forecast — a forecast with a mean-square forecast error smaller than the mean-square forecast error of the driftless random walk, and with robust out-of-sample test statistics over different forecast windows.⁷ In Section 3.3, we introduce the out-of-sample tests we consider — the asymptotic Clark-West and the bootstrapped Diebold-Mariano/West, Theil's U, Clark-West and Clark-McCracken test statistics. We discuss the differences between the alternative test statistics, the most important of which is that in cases of forecast bias, the new nested model tests should be interpreted as testing against the null hypothesis that the true model is a random walk, rather than as asking whether a random walk has a lower mean-square forecast error than the structural model (which is what the older Theil's U and Diebold-Mariano/West statistics test). These turn out to be quite different questions, although we also show that the newer nested model statistics can point to cases where it may be possible to improve on the random walk forecast by using it in combination with the structural model forecast. Nevertheless, finding an endogenous optimal combination may be a significant obstacle.

Section 3.4 tests the robustness of the apparent best results of the literature on short-horizon forecasting with respect to using alternative out-of-sample test statistics. The main studies reviewed are Gourinchas and Rey (2007) — an external balance model; Molodtsova and Papell (2009) — a heterogeneous symmetric Taylor rule model with smoothing; and Engel, Mark, and West (2007) — the monetary model. We conclude that in certain cases the popular Clark-West and Clark-McCracken test statistics are highly significant while the bootstrapped Theil's U and Diebold-Mariano/West are not which we attribute to the presence of forecast bias. Furthermore, in a couple of cases, the asymptotic Clark-West incorrectly chooses the structural model forecast over the random walk forecast for a different reason — the asymptotic Clark-West test seems to be oversized. In Section 3.5, we explore the robustness of the results of these same studies with respect to different forecast windows using a graphic approach which illustrates how the significance of the results is affected by perturbing the sample. We find that even those results that are robust to alternative out-of-sample test statistics are not robust when the forecaster considers alternative samples, with the external balance model of Gourinchas and Rey (2007) performing somewhat better than the rest of the specifications considered. This point strongly reinforces our conclusion from Section 3.4 that the results of the new models and specifications are not very robust.

Therefore, we attempt to improve upon existing panel specifications in Section 3.6 by taking into account

⁷ "Forecast window" refers to the part of the sample for which forecasts are calculated. For example, if we have a sample of 120 quarters and the first forecast is based on 30 quarters, then the forecast window is 90 quarters.

persistent cross-country shocks using purchasing power parity as a fundamental. Similarly to the results in Section 3.4, our results point to a discrepancy between the old out-of-sample test statistics and the new out-of-sample tests for nested models. In Section 3.7 we present empirical evidence of how one can improve upon our results from Section 3.6 by correctly interpreting the new nested model tests and combining the structural model forecast and the random walk forecast. At first look, our results are not worse than the most prominent results of other existing short-horizon forecasting studies. Nevertheless, the fact that so much of the forecasting power comes from simply using a different time dummy effect forecast gives us pause in attributing too much of the success to macroeconomic models. Finally, we subject our pooled forecast specification to a robustness check with respect to alternative forecast windows and conclude that even our preferred forecasting procedure cannot consistently outperform the driftless random walk over different forecast windows.

3.2 Definition of a "Good" Exchange Rate Forecast

There are various criteria for identifying a "good" forecast.⁸ One of the most widely used measures, popularized in the exchange rate literature by Meese and Rogoff (1983a), is the minimum mean-square forecast error (MSFE) approach, also known as the MSFE – dominance approach.⁹ The goal of this approach is to obtain a model whose MSFE is significantly smaller than that of the random walk model. As Clements and Hendry (2001) suggest, minimum MSFE has become the standard measure of forecast accuracy due to its intuitive interpretation and broad applicability (pp. 9). Another more stringent criterion, introduced by Chong and Hendry (1986), Clements and Hendry (1993), and Harvey, Leybourne, and Newbold (1998) is MSFE encompassing for nested models, which tests whether the structural model encompasses the random walk model. If it does not, then the information provided by the additional explanatory variables does not improve the forecast. MSFE encompassing is more stringent than MSFE dominance, since the latter is a

⁸We use the terms "forecast" and "out-of-sample forecast" interchangeably. In order for a forecast to be an out-of-sample forecast, a forecast in period t needs to be a function only of information available in period $t - k$ where the k is the forecast horizon. (For example, if $k = 1$ then we are forecasting one period ahead.)

When evaluating the performance of a structural model out-of-sample, we need to be able to compare the forecast produced by the model to the actual realized value of the series we want to forecast. As a result, we split the sample in two – in-sample portion and out-of-sample portion. We run a regression using the in-sample portion and calculate a forecast using the parameters from this regression. We can calculate the forecasts using a recursive or a rolling specification.

The recursive method adds one more observation to the in-sample portion for each additional period forecast. For example, if the first forecast is based on the first R observations, then the second forecast is based on the first $R+1$ observations, etc. In contrast, the rolling specification method preserves the original sample size throughout; hence, the first forecast is based on observations from 1 to R , the second on observations from 2 to $R+1$, and so on.

⁹Another less popular technique, which our paper does not address, uses the "direction of change" criterion. This criterion, of course, can end up selecting a model which performs well in predicting small changes but poorly at predicting major ones.

necessary but not sufficient condition for the former. MSFE encompassing also ensures that pooling the competing forecasts cannot produce a forecast with a smaller MSFE than the two nested models considered. A third criterion, robustness over different forecast windows, measures how consistently the structural model outperforms the random walk during different periods of time.

In what follows, we focus first on the minimum MSFE criterion, and afterwards look at robustness over different forecast windows.¹⁰

3.3 Minimum Mean-Square Forecast Error Tests: Theil's U (TU), Diebold–Mariano/West (DMW), Clark – West (CW), Clark – McCracken (ENC-NEW)

Before we address the performance of the structural models, we need to revisit the most widely used test statistics in the literature. Until Clark and West (2006), Clark and West (2007), Clark and McCracken (2001) and Clark and McCracken (2005) introduced their tests for nested models, the Theil's U and the Diebold-Mariano/West test statistics were the preferred minimum MSFE out-of-sample test statistics used in the exchange rate forecasting literature. In this paper, we consider the bootstrapped¹¹ version of both the new and old out-of-sample test statistics (DMW, TU, CW and ENC-NEW) and the asymptotic version

¹⁰We choose not to consider the encompassing criterion for a number of reasons. First, forecast encompassing, defined as the structural model encompassing the random walk, is not widely used in the exchange rate forecasting literature. Second, it is considered a more stringent criterion than MSFE dominance. Third, as Marcellino (2000) points out, the standard encompassing tests may not imply MSFE dominance in the presence of forecast bias. This point is somewhat related to our theoretical argument that the Clark-West and Clark-McCracken out-of-sample tests cannot be always interpreted as minimum MSFE tests in the presence of forecast bias (See Appendix and Section 3.3 for details).

¹¹All of the empirical results presented in the following sections are based on a bootstrap similar to the one used by Mark and Sul (2001). The main difference between our bootstrap and Mark and Sul (2001)'s bootstrap is that we use a "semi-parametric" while they use a "parametric" bootstrap and we estimate the error-correction equations using country-specific OLS-regressions rather than seemingly unrelated regressions (SURs) (Note that the "semi-parametric" bootstrap we use is closer in its nature to the "parametric" rather than the "non-parametric" bootstrap. For details on the bootstrap see Appendix).

We choose to use a "semi-parametric" rather than "non-parametric" bootstrap as our preferred bootstrap for a number of reasons. First, based on simulations, Berkowitz and Kilian (2000) argue in their paper "Recent Developments in Bootstrapping Time Series" that when bootstrapping time series, the "parametric" and "semi-parametric" bootstrap outperforms "non-parametric" bootstrap procedures.

Second, the exchange rate forecasting literature provides prolific evidence of the importance of preserving the cointegration between the fundamental and the exchange rate when estimating the exchange rate forecast equation (for example see Kilian (1999) and Mark and Sul (2001)). And as Berkowitz and Kilian (2000) point out

"While nonparametric bootstrap methods can easily deal with I(1) processes, there are no theoretical results to show that nonparametric resampling preserves cointegration relationships in the data. In fact, cointegration itself may be viewed as a parametric notion. Thus, if the data are known to be cointegrated, parametric methods are preferable (pp. 28)."

For further discussion of cointegration and bootstrapping see Li and Maddala (1997) and G. S. Maddala (1998) (pp. 333-336). For completeness sake, we try a number of non-parametric bootstraps such as the wild bootstrap and the block bootstrap but, not surprisingly, their performance is fairly weak and obvious mis-specification problems are present.

of the CW. (For a detailed description of how we calculate each test statistic and how we test statistical significance see Appendix.)

Among the asymptotic test statistics, we focus only on the CW because it has become one of the most popular out-of-sample test statistics for nested models.¹² Furthermore, as we point in the Appendix, the asymptotic versions of the DMW, TU and ENC-NEW have significant shortcomings or are non-tractable. One of the main reasons for the popularity of the asymptotic CW is that the alternative – the use of a bootstrap – is still considered by some researchers computationally cumbersome and difficult to implement.

In this paper we argue that while using the asymptotic CW might seem appealing due to its straightforward application, it is important that one checks the robustness of the results using either the bootstrapped DMW or the bootstrapped TU. The rationale follows.

3.3.1 CW/ENC-NEW – Not Always Minimum MSFE Tests

One of the main problems related to using the new tests for nested models (CW and ENC-NEW) as the *main and only* out-of-sample test statistics relates to the fact that they cannot be always interpreted as minimum MSFE tests such as the TU and the DMW. In the Appendix we prove that in the presence of forecast bias¹³ the CW/ENC-NEW and the DMW do not necessarily test the same null hypothesis; the CW and ENC-NEW test whether the exchange rate is a random walk, whereas TU and DMW test whether the random walk model and the structural model have equal MSFEs. These questions are not equivalent; if the true model is something other than a random walk, one can still perfectly well ask if the random walk model produces a lower mean-square forecast error.¹⁴ However, a significant CW/ENC-NEW and an insignificant bootstrapped TU/DMW can still provide potentially useful information as we show in sections 3.6 and 3.7. It implies that, *in theory*, one can pool the forecasts of the structural model and the random walk to produce a combined forecast that outperforms the random walk in terms of MSFE (See Appendix for proof). However,

¹²The list of studies which test statistical significance using the asymptotic CW includes Engel, Mark, and West (2007), Gourinchas and Rey (2007), Molodtsova and Papell (2009), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2010), Rapach, Strauss, and Wohar (2007), Sellin (2007), Alquist and Chinn (2008), Cerra and Saxena (2010), Sellin (2007), Groen (2007), Giacomini and Rossi (2010), Ardic, Ergin, and Senol (2008).

¹³Note that by forecast bias we imply only "scale" bias (see footnote 3 for details).

¹⁴If one tests the explanatory power of the structural model *in-sample* using an ordinary least square (OLS) regression, testing whether the exchange rate is a random walk (testing whether the coefficient in front of the structural model fundamental, b , equals zero) is equivalent to testing whether the random walk has mean square error (MSE) smaller than the MSE of the structural model because OLS minimizes the MSE. However, as the proof of Proposition C.1.1 in the Appendix shows, in the *out-of-sample* case, due to potential forecast bias resulting from forecast uncertainty, testing whether b equals zero is not the same as testing whether the MSFE of the random walk is smaller than the MSFE of the structural model.

finding an endogenous way of determining this optimal weight has proven to be a challenge (See Section 3.7 for further discussion).

3.3.2 The Asymptotics of CW Are Well Defined Only in the Rolling Case

Another problem related to the popular Clark-West out-of-sample test statistic is that the asymptotics of CW are well-defined only when we use the test statistic in a rolling framework, where the size of the in-sample portion of the series is kept fixed. For the recursive case (which comprises the majority of exchange rate forecast specifications in the literature), where the in-sample size varies, one has to use simulated critical values based on Brownian motion approximation of the limiting distribution of the CW test statistic.¹⁵ Throughout the paper, the term "asymptotic CW" refers to both the rolling and the recursive case. However, one should keep in mind that in the recursive case the asymptotic distribution of CW is approximated.

3.3.3 Bootstrapped Tests Are Relatively Better Sized and More Powerful

Finally, assuming that the bootstrap has been specified correctly, in most specifications, the bootstrapped DMW and TU out-of-sample tests are more powerful and better sized than the asymptotic CW.¹⁶ Moreover, new research on time series bootstrapping (see for example Li and Maddala (1997), Berkowitz and Kilian (2000), Kilian (1999) and Mark and Sul (2001)) and significant improvements in computational power have made the bootstrap an attractive alternative to asymptotic inference.¹⁷

¹⁵As the authors emphasize, no formal proof is presented that the critical values suggested are appropriate for all forecast specifications (Clark and West (2007), pp. 298).

¹⁶See the "Not for Publication Appendices" of Clark and West (2006) and Clark and West (2007) that can be found on Kenneth West's website. (Note that in the 2006 Appendix both DGP 1 and 2 are relevant for exchange rate forecasting while in the 2007 Appendix only DGP 1 is of interest.) Regarding comparison between the bootstrapped TU and DMW, see Clark and McCracken (2005).

The concepts of size and power are key to understanding the differences between the alternative out-of-sample test statistics. They are properties of both the asymptotic and bootstrapped tests. The size of a test statistic is defined as the test's probability of rejecting the null hypothesis if the null is true. If the researcher chooses to use a significance level of 10%, an under-sized (over-sized) test statistic would tend to reject the null hypothesis in less (more) than 10% of the cases. If a test statistic is over-sized, it might incorrectly detect statistical significance if such does not exist and if it is under-sized – incorrectly reject the alternative. The power of a test statistic is defined as the test's probability of correctly rejecting the null hypothesis for a given level of statistical significance. The size and power of a test statistic are inversely related.

In the Appendix of Clark and West (2006), it is not immediately obvious why the bootstrapped DMW has greater power than the asymptotic CW because the authors report size-adjusted power rather than raw power. The main difference between the two is that only raw power is of any practical importance since in order to adjust for size distortions, the size-adjusted power is based on a CW test statistic which uses data specific critical values obtained via Monte Carlo simulation. Since few, if any, researchers would choose this alternative, the raw power is what one is mainly interested in. Given that the size-adjusted power of CW is similar to that of the bootstrapped DMW, the raw power of CW will be smaller than the raw power of the bootstrapped DMW. This is the case because the CW is somewhat undersized while the bootstrapped DMW seems adequately sized and as we already explained the size and power of a test statistic are inversely related.

Finally, according to the simulation evidence in Clark and McCracken (2005), we would expect the bootstrapped TU to be more powerful than the bootstrapped DMW. (Note that in their paper the authors discuss the power of the MSE-F rather than the TU but the two tests are very similar). Since the bootstrapped DWM is more powerful than the asymptotic CW, we would expect that the bootstrapped TU is more powerful than the asymptotic CW as well.

¹⁷See Appendix for further discussion on time series bootstrapping.

As a result, we treat the bootstrapped DMW and TU in this paper as our preferred out-of-sample test statistics. In what follows, first we test the robustness of the results of the most popular exchange rate forecasting models and specifications with respect to alternative out-of-sample tests. Second, we concentrate on the robustness of these same specifications with respect to using different sub-samples.

3.4 Robustness With Respect to Alternative Test Statistics

In this section, we evaluate the robustness of the best results of Gourinchas and Rey (2007), Molodtsova and Papell (2009) and Engel, Mark, and West (2007) with respect to alternative test statistics (bootstrapped CW, ENC-NEW, DMW and TU). These studies all feature the asymptotic CW as their main out-of-sample test statistic and conclude that for a number of countries, structural models outperform the driftless random walk for forecasts one period ahead. While Engel, Mark, and West (2007) attribute their success to the power of panel models, Molodtsova and Papell (2009) and Gourinchas and Rey (2007) find success using new structural models. While we concentrate our attention on these three prominent studies with fairly positive results, we believe that the implications of our findings are generalizable to the rest of the new literature on short-horizon exchange rate forecasting.

We conclude that forecast bias is a serious problem which in certain specifications leads to a significant discrepancy between the CW/ENC-NEW and DMW/TU. Furthermore, in a couple of cases, the asymptotic CW is oversized. As a result of both of these issues, some of the results of the literature are overly optimistic and potentially misleading.

3.4.1 Engel, Mark and West (2007) — The Monetary Model

The implementation of a panel forecast specification is one of the key additions to the exchange rate forecasting literature which allows Engel, Mark, and West (2007) to find limited forecastability of the exchange rate change one quarter ahead.¹⁸ The study finds that for 5 out of 18 currencies, the monetary model outperforms the driftless random walk. While recognizing this success as modest, the authors note that their results appear notably more positive than the norm in the literature.¹⁹

¹⁸The study of Engel, Mark, and West (2007) builds on Engel and West (2005).

¹⁹The majority of the recent panel specification papers find strong support for the forecasting power of the monetary model in both long and short horizons (see Mark and Sul (2001), Rapach and Wohar (2002), Engel, Mark, and West (2007) and Groen (2005)). However, at the same time, the theoretical validity of the monetary specification has been widely criticized. The criticism of the monetary model centers around its assumptions that both purchasing power parity and uncovered interest

The forecasting specification Engel, Mark, and West (2007) apply is straightforward. The forecast variable is the nominal exchange rate change, where s_t is the natural log of the exchange rate measured in foreign currency per one unit of the base currency (in this case US dollars). Define $\Delta s_{i,t+1} = s_{i,t+1} - s_{i,t}$ and the forecast is one period ahead. Then the panel forecast equation can be expressed as

$$\Delta s_{i,t+1} = \alpha_i + \theta_t + \beta z_{i,t} + \varepsilon_{i,t+1}. \quad (3.4.1)$$

where, in this case, $z_{i,t}$ stands for the deviation of the exchange rate from an equilibrium value. $z_{i,t}$ is determined by the monetary model fundamental

$$z_{i,t} = m_{i,t} - m_t^* - \varphi(y_{i,t} - y_t^*) - s_{i,t}. \quad (3.4.2)$$

Above, i is a country-specific index, α_i stands for country-specific effects, θ_t - for time specific effects and $\varepsilon_{i,t+1}$ is the innovation term. The $(*)$ represents the base country (the *US*), $(m_{i,t} - m_t^*)$ is the relative money supply, $(y_{i,t} - y_t^*)$ is the relative income level and φ is assumed to be one. Note that all the variables are in natural logs. We use the driftless random walk, expressed as $\Delta s_{i,t+1} = v_{i,t+1}$ (where $v_{i,t+1}$ is the innovation term of the driftless random walk model) as a benchmark which would ensure that the structural model is compared to the best known alternative.²⁰

Engel, Mark, and West (2007) estimate equation (3.4.1) using recursive *OLS* regressions. They calculate the exchange rate change forecast using the following equations.

$$\text{Structural Model : } \Delta \hat{s}_{i,t+1} = \hat{\alpha}_i + \hat{\theta}_{t+1} + \hat{\beta} z_{i,t+1}$$

$$\text{Driftless Random Walk Model : } \Delta \hat{s}_{i,t+1} = 0$$

where the time dummy for period $t + 1$ is calculated as $\hat{\theta}_{t+1} = \frac{1}{t} \sum_{j=1}^t \hat{\theta}_j$. Engel, Mark, and West (2007) sample extends Mark and Sul (2001)'s data set up to 2005Q4. The exchange rates of the Eurozone countries post 1999 are normalized in a way that they differ from each other only by a constant.²¹ This implies that

parity hold. However, these assumptions are not unequivocally supported by empirical evidence (Engel (1996)). Furthermore, there is a debate on how one defines the money supply, the stability of the money equation (Friedman and Kuttner (1992)) and whether money has any relevance for economic decision making such as monetary policy.

²⁰Engel, Mark, and West (2007) compare the forecasts of the monetary model to both the random walk with drift and without drift. However, they note that the driftless random walk outperforms the random walk with drift. All of the studies we are aware of that compare the driftless random walk to the random walk with drift, find the driftless random walk to be a better forecaster (see Engel and Hamilton (1990) and Engel, Mark, and West (2007)).

²¹For example, the normalization for France post 1999 will be simply franc/euro times euro/dollar where the franc/euro is the peg used to fix the French franc to the euro in 1999.

post 1999, Engel, Mark, and West (2007) specification is essentially forecasting the same exchange rate - the Euro - using different country specific monetary fundamentals. For further details on the specification and for data set sources refer to Engel, Mark, and West (2007).²²

We test the robustness of their results with respect to different test statistics. In Table 3.4.1, we reproduce the monetary model results but rather than just report the asymptotic CW test statistic, we also report the bootstrapped p-values of the DMW, TU, CW and ENC-NEW. If we test statistical significance via the bootstrapped DMW and TU test statistics, the p-value is less than 10% for only 4 out of 18 cases.²³ These results are confirmed by the bootstrapped CW. However, Greece stands out as an example where the asymptotic CW is statistically significant while the bootstrapped CW is not, suggesting that the asymptotic CW is oversized.²⁴

²²We use Engel, Mark, and West (2007)'s data except for exchange rates which are from the IFS data set. The bootstrap procedure is similar to Mark and Sul (2001) and assume no unit root of the monetary fundamental. (For details on the bootstrap used see Appendix.)

²³Another way of testing for robustness, which we do not pursue in this paper, is by estimating to what extent the positive results could be attributed to the large number of specifications tested. For instance, the test statistic introduced by McCracken and Sapp (2005) tests whether the number of successful forecasts can be attributed solely to the large number of specifications and models estimated by the researcher. If we were to calculate McCracken and Sapp (2005)'s test statistic, the results might have been even less favorable for the structural models.

²⁴In the case of Greece, the asymptotic CW performs so poorly because the DMW is largely oversized and, as a result, the asymptotic CW is even more over-sized (see Appendix for clarification on the difference between CW and DMW). The mean of the bootstrapped DMW histogram is 1.3 (and it should have been 0 if no size problem was present). As is apparent from the results in Table 3.4.1, in outlier cases like Greece, the asymptotics fail while the bootstrap, if properly specified, seems to be still reliable.

Table 3.4.1

Mark, Engel and West (2007)
Monetary Model Vs Random Walk with No Drift; One Quarter Ahead

	CW [^]	Reproduced Results							
		CW	P-value	TU	P-value	DMW	P-value	ENC-NEW	P-value
UK	0.684	0.624	0.25	1.001	0.262	-0.063	0.254	0.739	0.251
Austria	1.966	1.854	0.04	0.984	0.040	1.315	0.015	2.164	0.110
Belgium	1.199	0.972	0.13	1.001	0.327	-0.041	0.292	1.653	0.034
Denmark	0.259	0.178	0.372	1.009	0.657	-0.553	0.459	0.257	0.340
France	0.706	0.545	0.23	1.001	0.228	-0.050	0.225	0.543	0.237
Germany	1.855	1.711	0.056	0.986	0.048	0.924	0.065	2.408	0.080
Netherlands	1.651	1.400	0.084	0.990	0.071	0.922	0.060	1.375	0.144
Canada	-0.942	-0.936	0.881	1.161	0.970	-2.348	0.906	-4.240	0.988
Japan	1.094	0.671	0.439	0.999	0.366	0.038	0.364	0.873	0.425
Finland	0.648	0.696	0.262	1.004	0.420	-0.156	0.339	1.463	0.154
Greece	2.509	2.501	0.704	1.004	0.906	-0.085	0.899	####	0.450
Spain	0.711	0.699	0.592	1.027	0.916	-0.806	0.799	2.091	0.343
Australia	0.787	0.727	0.343	1.026	0.555	-0.914	0.479	1.869	0.303
Italy	0.733	0.557	0.519	1.015	0.764	-0.487	0.575	1.525	0.401
Switzerland	1.965	1.985	0.06	0.986	0.082	1.409	0.019	1.912	0.197
Korea	0.853	0.847	0.385	0.997	0.252	0.118	0.306	1.972	0.188
Norway	0.645	0.271	0.327	1.005	0.426	-0.402	0.358	0.296	0.317
Sweden	1.100	1.030	0.225	0.993	0.163	0.466	0.174	1.372	0.212

[^]Results provided by Charles Engel using a corrected data set

The benchmark is a random walk without drift; Quarterly data ranging from 1973Q1 to 2005Q4; First Forecast: 1983Q1; The p-value is the bootstrapped version of the respective test statistic. Bootstrap based on 1000 iterations; Bold p-values imply statistical significance of 10% or less; Bold Theil's U values represent Theil's U ≤ 1; Bold CW values represent statistical significance of 10% (above 1.282) using Clark and West's (2007) simulated critical values.

3.4.2 Molodtsova and Papell (2009) — Heterogeneous Symmetric Taylor Rule with Smoothing

In addition to the improvements produced by the panel specification, the introduction of the Taylor rule as a structural fundamental has also seemed to yield improved forecasts. The specification which produces best forecasting results estimates country specific coefficients on both inflation and the output gap. Furthermore, Molodtsova and Papell (2009) assume that interest rates adjust only partially to its target and, as a result, lagged interest rates are included in the specification which represent the so-called smoothing effect. Using only single-country equations and the asymptotic CW test statistic, Molodtsova and Papell (2009) conclude

that the Taylor rule outperforms the driftless random walk for 10 out of 12 currencies for forecasts one period ahead. Molodtsova and Papell (2009) specify the fundamental, z_t , as

$$z_t = \alpha_1 \pi_t + \alpha_2 \pi_t^* + \alpha_3 y_t^{gap} + \alpha_4 y_t^{gap*} + \alpha_5 i_{t-1} + \alpha_6 i_{t-1}^* \quad (3.4.3)$$

where π is the inflation rate, i is the interest rate and y^{gap} is the output gap defined as the deviation of an industrial production index from a linear trend. We substitute equation (3.4.3) in equation (3.4.1) and estimate equation (3.4.1) in a single equation framework using monthly data.²⁵ Molodtsova and Papell (2009) refer to specification (3.4.3) as the heterogeneous symmetric Taylor rule with smoothing.²⁶ More information regarding the specification and data sources is provided in Molodtsova and Papell (2009).

Similarly to the monetary model specification of Engel, Mark, and West (2007), we replicate Molodtsova and Papell (2009)'s results and compute not only the asymptotic CW, but also the bootstrapped TU, DMW, CW and ENC-NEW.²⁷ Table 3.4.2 reports Molodtsova and Papell (2009) results as presented in their paper and our attempt to replicate them using their methodology and data set.

There is a striking difference between both the bootstrapped and asymptotic CW and the bootstrapped ENC-NEW, on the one hand, and the bootstrapped TU and DMW on the other hand. While CW and ENC-NEW are significant in as many as 10 out of 12 cases, the TU is not significant for any of the countries and DMW is significant only for Canada. We explain this discrepancy with the presence of severe forecast bias in which case the CW and ENC-NEW cannot be interpreted as minimum MSFE tests and they do not test the same null hypothesis as TU and DMW.²⁸ In the case of Switzerland, the bootstrapped CW is insignificant while the asymptotic CW is significant, again, suggesting that the asymptotic CW might be oversized in certain cases.

²⁵Single-equation framework implies that there are no time dummy effects.

²⁶The estimation method is a rolling regression specification with a rolling window of 120 months.

²⁷The data set was provided to us by the authors and it is also available on David Papell's website. For the bootstrap, similarly to the bootstrap used to replicate the results of Engel, Mark, and West (2007)'s study, we use similar to Mark and Sul (2001)'s procedure. We assume that the inflation rates, the interest rates and the output gaps do not have unit roots. (For details on the bootstrap used see Appendix.)

²⁸A regression of the observed exchange rate change on the forecast series and no constant produces a coefficient less than or close to 0.5 for all 10 countries where CW and ENC-NEW are significant. (If no "scale" forecast bias was present, the coefficient should have been close to 1.) This is what we would expect in cases of severe "scale" forecast bias which can lead to CW and ENC-NEW not testing the same null as TU and DMW.

Table 3.4.2

Molodtsova and Papell (2008)
Heterogeneous Symmetric Taylor Rule with Smoothing Vs Random Walk with No Drift; One Month Ahead

	value asympto	value asymptot	value bootstrap	TU	P- value	DMW	P- value	ENC- NEW	P- value
UK	0.020	0.027	0.027	1.051	1.000	-1.740	0.678	14.662	0.001
Denmark	0.069	0.067	0.045	1.025	0.992	-1.231	0.397	8.067	0.013
France	0.024	0.019	0.007	1.040	0.998	-1.260	0.557	11.312	0.001
Germany	0.066	0.066	0.077	1.036	0.997	-1.130	0.548	8.458	0.016
Netherland	0.036	0.035	0.040	1.040	1.000	-1.304	0.613	9.604	0.012
Canada	0.008	0.008	0.008	1.006	0.174	-0.261	0.078	15.025	0.003
Japan	0.019	0.019	0.071	1.018	0.912	-0.723	0.367	14.152	0.008
Australia	0.015	0.013	0.039	1.024	0.972	-0.895	0.360	15.130	0.004
Italy	0.002	0.002	0.039	0.995	0.264	0.168	0.327	18.240	0.003
Switzerland	0.094	0.094	0.153	1.068	1.000	-2.198	0.910	9.151	0.021
Sweden	0.678	0.674	0.667	1.098	1.000	-1.261	0.494	-5.897	1.000
Portugal	0.985	0.985	0.985	1.127	1.000	-3.329	0.999	-4.464	1.000

^ Results as reported in Molodtsova and Papell (2008)

Single equation, monthly data. Since Molodtsova and Papell (2008) use rolling regressions, the asymptotic CW p-values are calculated under the assuming of normality; The TU, ENC-NEW and DMW p-values and the CW bootstrap p-value are based on a bootstrap (1000 iterations); Bold Theil's U values represent Theil's U ≤ 1;

3.4.3 Gourinchas and Rey (2007) — External Balance Model

Another important study that claims to successfully forecast exchange rates one period ahead is Gourinchas and Rey (2007). The authors introduce a new external balance model which isolates long- term effects by defining an external balance variable as a function of de-trended foreign assets and liabilities, exports and imports. Gourinchas and Rey (2007) find that their external balance measure is superior to those previously used in the literature on external balance specifications since it takes into account capital gains and losses on the net foreign asset position, in addition to the trade balance. Gourinchas and Rey (2007) argue that their external balance variable successfully forecasts both the trade and FDI-weighted dollar one quarter ahead.

We can write Gourinchas and Rey (2007)'s external balance fundamental as

$$z_t = |\mu_t^a|\epsilon_t^a - |\mu_t^l|\epsilon_t^l + |\mu_t^x|\epsilon_t^x - |\mu_t^m|\epsilon_t^m \quad (3.4.4)$$

where μ_t^a , μ_t^l , μ_t^x and μ_t^m are time-varying weights – a function of the Hodrick-Prescott-filtered trends of assets, liabilities, exports and imports – while ϵ_t^a , ϵ_t^l , ϵ_t^x and ϵ_t^m represent the log deviation of assets, liabilities, exports and imports from Hodrick-Prescott-filtered trends. Equation (3.4.4) is substituted into equation

(3.4.1) and the authors estimate equation (3.4.1) for the trade-weighted and the FDI-weighted exchange rate separately in a single equation framework.²⁹ Gourinchas and Rey (2007) assume that the time-varying weights converge asymptotically and use fixed weights for the calculation of their forecasts. Further details on the specification and the data set used are provided in Gourinchas and Rey (2007). In Table 3.4.3, we reproduce their results using their data set and similar methodology.³⁰ One can observe highly significant asymptotic CW, bootstrapped TU, DMW, CW and ENC-NEW test statistics. However, the seemingly strong result is overturned, to an extent, when checking for robustness with respect to alternative time periods in the following section.

Table 3.4.3

Gourinchas and Rey (2007)									
External Balance Model Vs Random Walk with No Drift; One Quarter Ahead									
Reported by G&R		Reproduced Results							
CW		CW	P-value	TU	P-value	DMW	P-value	ENC-NEW	P-value
Trade Weighted Exch Rate	2.690	2.684	0.005	0.974	0.003	0.657	0.013	11.774	0.001
FDI Weighted Exch Rate	2.196	2.191	0.006	0.980	0.005	0.821	0.019	5.780	0.002

FDI - Weighted exch rate - rolling regression where the rolling window is 105 quarters; first forecast: 1978 Q3
Trade - Weighted exch rate - recursive regression; first forecast: 1978 Q2
The p-value is the bootstrapped version of the respective test statistic. Bootstrap based on 1000 iterations; Bold p-values imply statistical significance of 10% or less; Bold Theil's U values represent Theil's U <= 1; Bold CW values represent statistical significance of 10% (above 1.282) using Clark and West's (2007) simulated critical values.

Summary of Test Statistic Robustness

We looked at each one of the three major studies which find forecastability one period ahead and concluded that when one considers the robustness of the results with respect to alternative test statistics, the results of Molodtsova and Papell (2009) fluctuate significantly due to the presence of forecast bias. The results of Engel, Mark, and West (2007) are somewhat less spectacular as a result of one outlier where the asymptotic CW is severely oversized. Finally, we conclude that the results of Gourinchas and Rey (2007) remain robust to the test statistic considered. Now we turn to our second main issue – which of the results are robust over

²⁹Note that the authors claim to be using a 105 quarter rolling window. However, a closer look at their code shows that they use 105 quarter rolling window for the forecasts of the FDI-traded dollar and a recursive specification for the trade-weighted dollar. We calculate the forecast both ways – in a recursive and rolling framework – and the results do not change substantially.

³⁰We are grateful to the authors for providing us with their code and data set. Note that in Table 3.4.3 we report the CW test statistic which is calculated as $CW = \frac{P^{0.5}\hat{d}}{\sqrt{\Omega^{\hat{d}}}}$ where \hat{d} is defined in equation (C.1.1) in Appendix, while Gourinchas and Rey (2007) report \hat{d} in their paper.

As before, we use a bootstrap procedure similar to Mark and Sul (2001) and assume no unit root of the external balance variable. (For details on the bootstrap used see Appendix.)

different periods of time.

3.5 Robustness with Respect to Different Forecast Windows

In addition to testing the robustness of the results with respect to different out-of-sample test statistics, we also test the robustness of the results of Molodtsova and Papell (2009), Engel, Mark, and West (2007) and Gourinchas and Rey (2007) via varying the forecast window. This is another important test of how consistently reliable the forecast is.³¹

We find that the structural models do not produce consistently better forecasts than the driftless random walk over different sample periods for the three studies reviewed. However, it seems that the monetary model and the Taylor rule model forecast the exchange rate better than the random walk during the 1980s while Gourinchas and Rey (2007)'s external balance model consistently outperforms the driftless random walk in the 1990s and the 2000s.

3.5.1 Engel, Mark and West (2007)

Figure 3.5.1 illustrates an approach to testing robustness to different sample periods in the context of Engel, Mark, and West (2007)'s monetary model. Figure 3.5.1 plots the bootstrapped TU (Theil's U) p-value on the y-axis and the starting date of the recursion on the x-axis (the first date for which a forecast is calculated). We plot only the bilateral exchange rates for which we find forecastability for a large number of forecast windows in order to make the graph legible. For similar reasons, we report only the results for Germany as a proxy for the Eurozone countries.

The way Figure 3.5.1 should be interpreted is the following. For example, the TU p-value associated with 1984Q4 for a given country implies that the TU p-value is calculated using the forecast window from 1984Q4 to 2005Q4 (the end of the sample). If the TU p-value is below 0.1, we consider the result statistically significant at 10 percent. In order for a result to be considered robust, we would expect that the TU p-value is below 0.1 for almost all of the plotted forecast windows. The graph shows that the monetary model

³¹Giacomini and Rossi (2010) is one of the few studies which attempts to formalize the issue of robustness over different forecast windows by developing appropriate test statistics.

is a relatively good forecaster of the Swiss franc and, to a lesser extent, of the Deutsche mark/euro.³² It is interesting to note that overall (when one considers the other current Eurozone countries as well), the monetary model performs relatively well in the 1980s and its performance deteriorates in the 1990s.

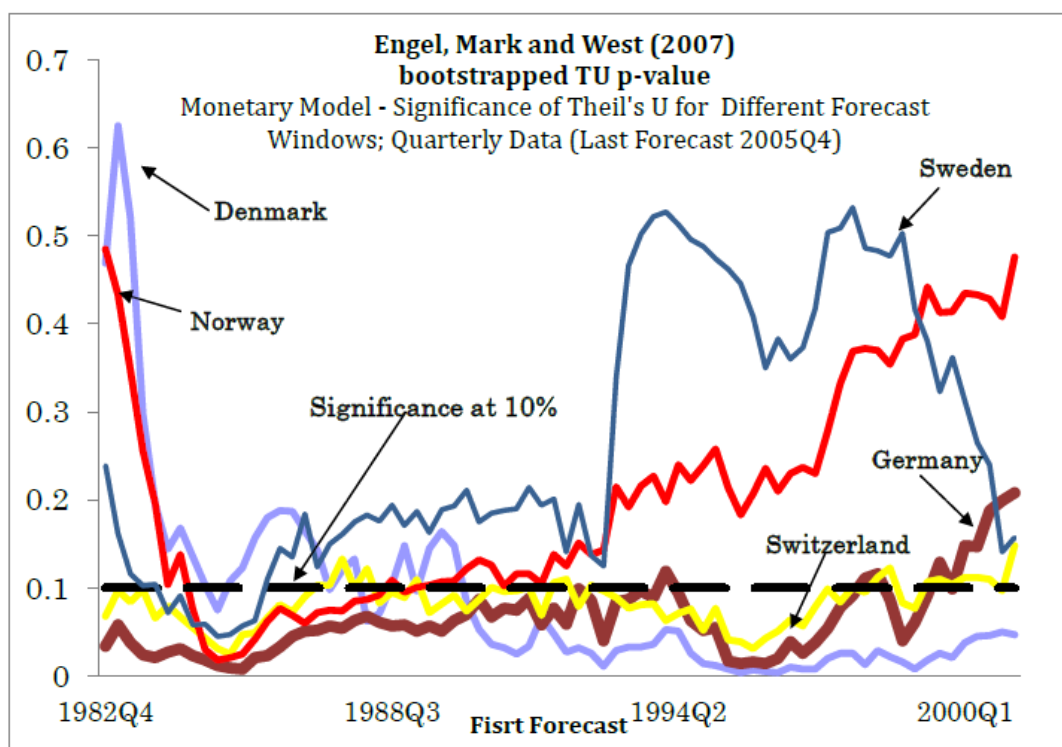


Figure 3.5.1

In conclusion, while at first look (considering one forecast window only), Engel, Mark, and West (2007)'s results seem encouraging, if one considers the robustness of the results over different forecast windows, they are less so.³³

3.5.2 Molodtsova and Papell (2009)

In a similar fashion, we evaluate Molodtsova and Papell (2009)'s heterogeneous symmetric Taylor rule results with smoothing for consistency over different forecast windows. Figure 3.5.2 depicts the robustness of Molodtsova and Papell (2009)'s results with respect to starting the rolling regression at a different date.

³²Keeping in mind that post-1999 the Deutsche mark transitions into the Euro, the fact that the TU test statistic for Germany becomes insignificant when we restrict the forecast window post year 2000 implies that while the monetary model was a relatively good forecaster of the Deutsche mark, this might not be the case for the Euro. However, with more euro data the result could change.

³³The results remain non-robust when one reproduces Figure 3.5.1 plotting the bootstrapped ENC-NEW/CW test statistic rather the bootstrapped TU p-value.

Only the countries for which the bootstrapped CW are significant are reported (nine out of twelve). Figure 3.5.2 clearly shows that the results are somewhat robust only for Canada – the only country for which the TU is below 0.1 for most of the forecast windows.³⁴

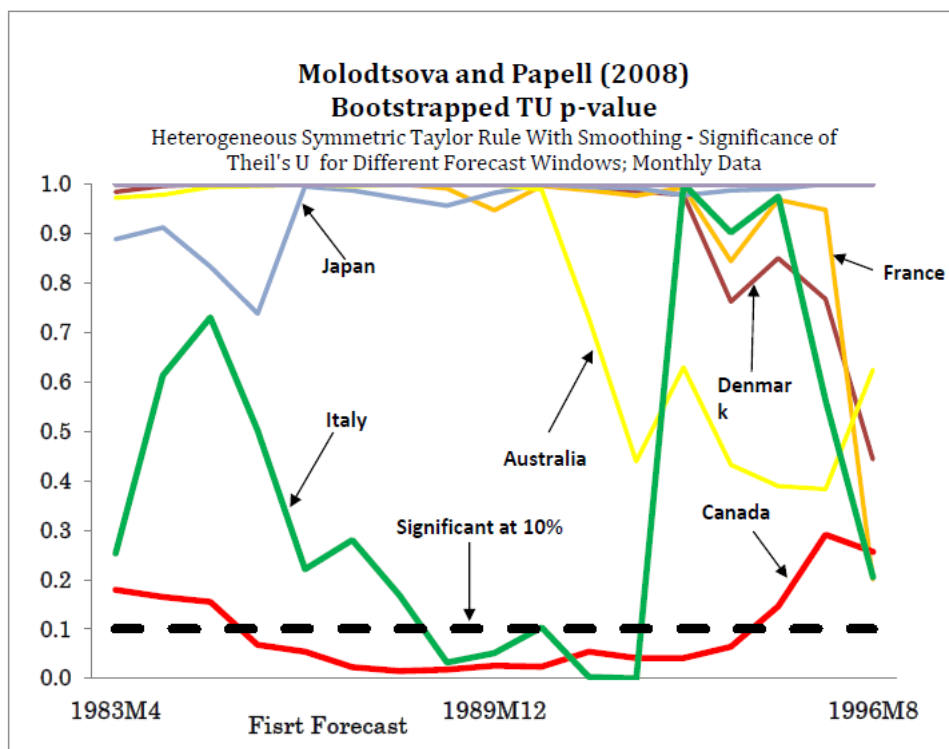


Figure 3.5.2

3.5.3 Gourinchas and Rey (2007)

Finally, in order to test to what extent the results of Gourinchas and Rey (2007)'s external balance model are robust to changing the forecast window, we report the p-value of the bootstrapped TU test statistic for different forecast windows in Figure 3.5.3. The forecasts are estimated using a recursive specification.³⁵ We report two approaches of estimating the external balance variable. The solid lines in Figure 3.5.3 represent

³⁴While interesting, the result for Canada is perhaps not that surprising given that Reinhart and Rogoff (2004) classify Canada as a limited flexibility exchange rate.

Even though, in Molodtsova and Papell (2009)'s specification, one cannot interpret the CW and ENC-NEW as minimum MSFE tests due to the presence of severe forecast bias, for completeness, we reproduce Figure 3.5.2 using the bootstrapped CW. The bootstrapped CW is significant in the early 1980s for the majority of the countries but not significant for the rest of the period (the only exceptions are Australia, Canada, Italy and Japan for which the bootstrapped CW is significant for the majority of forecast windows).

³⁵If we calculate the forecasts using a rolling window of 105 quarters (the rolling window Gourinchas and Rey (2007) state they use) rather than recursive regressions, the results do not change substantially. However, using recursive regressions allows us to check the robustness of the trade-weighted dollar for different forecast windows given the shorter range of the series. We also calculate time varying weights rather than impose constant weights which also seems to affect the final results only negligibly.

Gourinchas and Rey (2007)'s approach which estimates the external balance variable as a function of a discount constant, ρ , which is assumed to be 0.95 (a long-term value obtained using the entire sample). In order to test the robustness of Gourinchas and Rey (2007)'s results to alternative values of ρ , we calculate a time-varying discount rate (as opposed to a long-run value) using only the in-sample portion of the data. This approach is presented with dashed blue and red lines in Figure 3.5.3.

As before, we consider TU significant at 10 percent when the bootstrapped p-value is below the dashed black line. It is interesting to note that the FDI-weighted exchange rate performs relatively well for most of the periods with the exception of the late 1980s. The performance of the trade-weighted series is less impressive (especially when one considers the time-varying ρ approach which calculates the discount rate using only in-sample information). However, no matter whether one considers the trade-weighted or the FDI-weighted series, the external balance variable outperforms the driftless random walk in the 1990s and 2000s.³⁶

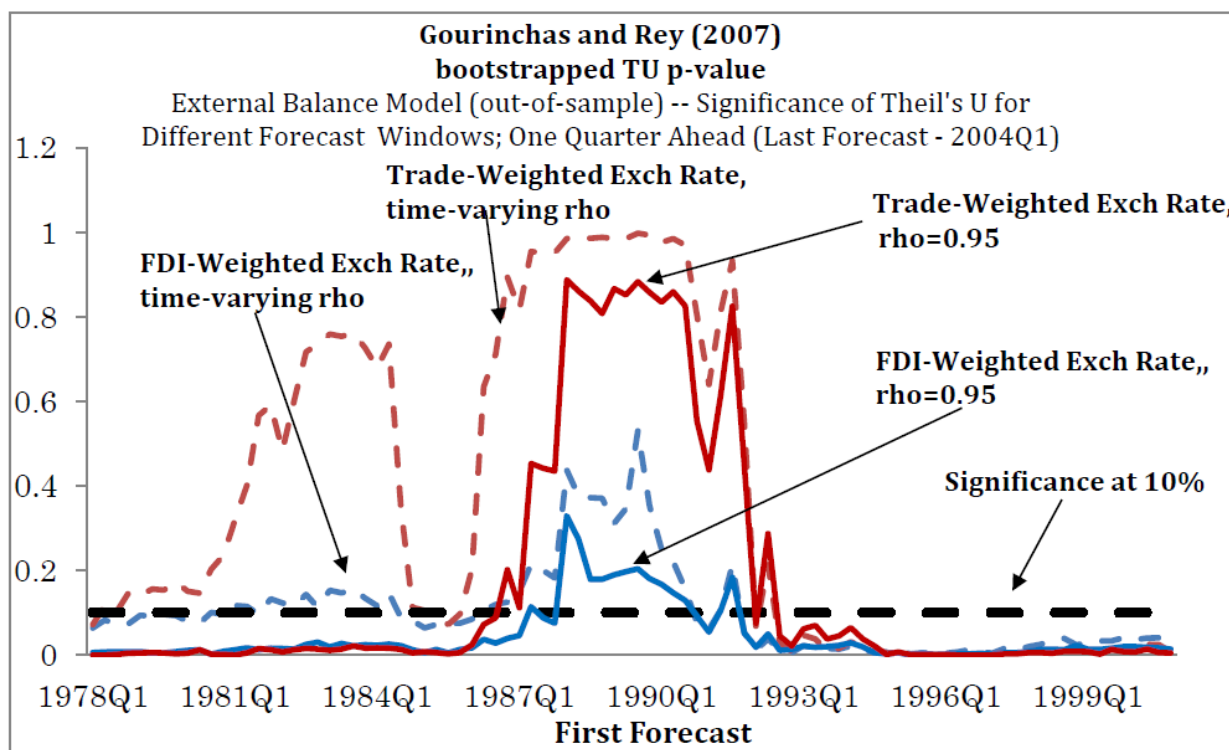


Figure 3.5.3

Summary of Robutness to Perturbation in Sample

After examining the performance of the monetary model, the external balance model and the heterogenous Taylor rule with smoothing over different periods of time, we conclude that Engel, Mark, and West (2007)'s

³⁶The results are similar if we consider the rest of the out-of-sample test statistics.

monetary model specification performs well in the 1980s but poorly in the more recent period. Molodtsova and Papell (2009)'s Taylor rule results, which were not robust to the use of the bootstrapped TU and DMW in Section 3.4, are also highly non-significant for all the periods considered. It seems that the performance of Gourinchas and Rey (2007)'s external balance variable is potentially more encouraging for the most recent period. However, one should be cautious in comparing the performance of structural models which forecast bilateral exchange rates with those which forecast weighted exchange rates, as the latter tend to be significantly less volatile.

3.6 Can One Do Better? The Importance of Common Cross-Country Shocks

In this section we try to improve upon the panel forecast specification applied by Mark and Sul (2001) and Engel, Mark, and West (2007) by incorporating persistent common cross-country shocks in the forecasts. These might include technology shocks, commodity price shocks, or factors related to the pace of globalization.

The basic forecast specification we use is the same as the one defined in equation (3.4.1). However, we define $z_{i,t}$, the deviation of the exchange rate from an equilibrium value, using the purchasing power parity model (PPP) rather than the monetary model.³⁷

$$z_{i,t} = p_{i,t} - p_t^* - s_{i,t} \quad (3.6.1)$$

Above, p is the natural log of the CPI and, as before, the $(*)$ represents the US. We substitute equation (3.6.1) into equation (3.4.1) and estimate equation (3.4.1) using recursive OLS panel regressions. The way we take into account potential persistent cross-country shocks is by forecasting the time dummy effect for period $t + 1$ differently from previous panel studies. Rather than forecast it simply as the average of the time-dummy coefficients for all the previous periods, as Engel, Mark, and West (2007) did, we forecast it as

³⁷The PPP specification is known to perform well at long horizons, but has been much less explored in looking at short-horizon nominal exchange rate forecasts. Engel, Mark, and West (2007) is the only study, we are aware of, which has explored the forecasting power of the PPP model at short horizons in a panel framework where the benchmark is the driftless random walk. Engel, Mark, and West (2007) find that for forecasts one period ahead, the PPP forecast is significantly better than the driftless random walk forecast only in 3 out of 18 cases.

We also perform the same type of forecasting exercise as in sections 3.4 and 3.5 using the monetary model, the Taylor rule and a new structural model based on the Backus - Smith optimal risk sharing condition model. Out of all the models we try, the PPP specification performs the best.

a simple average of the last 4 estimated time-dummy coefficients. Mathematically, the time dummy forecast can be defined as

$$\theta_{t+1} = \frac{1}{q} \sum_{j=t-q+1}^t \theta_j$$

where $q = 4$ when the data is quarterly.

Table 3.6 reports the results of the specification defined above.³⁸

Table 3.6

PPP Specification; One Quarter Ahead; Forecasts Incorporate Common Cross-Country Shocks

	CW	P-value	ENC-NEW	P-value	Theil's U	P-value	DMW	P-value
UK	0.423	0.326	1.343	0.242	1.072	0.991	-1.879	0.810
Denmark	1.469	0.072	3.884	0.040	1.008	0.268	-0.285	0.226
Germany	1.662	0.075	5.655	0.034	0.999	0.145	0.035	0.147
Canada	2.724	0.004	21.411	0.000	1.056	0.444	-0.665	0.142
Japan	1.809	0.090	5.592	0.056	1.004	0.280	-0.118	0.265
Australia	2.435	0.021	11.391	0.003	0.958	0.006	0.959	0.026
Switzerland	1.239	0.230	4.184	0.106	1.008	0.445	-0.250	0.385
Korea	1.870	0.059	6.634	0.015	0.975	0.007	0.712	0.056
Norway	0.944	0.155	2.363	0.115	1.033	0.741	-1.232	0.534
Sweden	2.215	0.030	5.398	0.035	0.999	0.155	0.028	0.157

Recursive specification; quarterly data; country and time dummies included; time dummy effect forecasted as the simple avg of estimated time dummies over last 4 quarters; first forecast 1983Q1; last forecast 2007Q1 (PPP); The p-value is the bootstrapped version of the respective test statistic. Bootstrap based on 1000 iterations; Bold p-values imply statistical significance of 10% or less; Bold Theil's U values represent Theil's U ≤ 1; Bold CW values represent statistical significance of 10% (above 1.282) using Clark and West's (2007) simulated critical values.

The results of Table 3.6 are very similar to the results in Molodtsova and Papell (2009) presented in Table 3.4.2. If we concentrate our attention only on the statistical significance of the bootstrapped TU and DMW test statistics, we notice that the results are significant only for Australia and Korea. However, when one calculates the CW and ENC-NEW out-of-sample test statistics, CW and ENC-NEW are significant for 7 out

³⁸The data source is the International Financial Statistics (IMF) (Data available upon request). Our data set consists of eleven countries: US, UK, Denmark, Germany, Canada, Japan, Australia, Switzerland, Korea, Norway and Sweden. We choose to proxy the Euro using the Deutsche mark series up to 1999 and the euro post 1999. The bootstrap specification is similar to Mark and Sul (2001) and the same as the bootstrap used in the literature review section (see Appendix for details).

of 10 countries. Note that the bootstrapped CW behaves similarly to the asymptotic CW. In combination with the fact that Clark and McCracken (2005), Clark and West (2006) and Clark and West (2007) conclude that the bootstrapped DMW and TU tend to be more powerful than the asymptotic CW and adequately sized, this implies that the discrepancy between the CW/ENC-NEW and TU/DMW cannot be attributed to different power and size. Furthermore, an investigation of the results indicates the presence of forecast bias.³⁹ As a result, the only explanation left for the discrepancy is that the two types of test statistics test different null hypotheses and cannot be used interchangeably – a point we prove and discuss further in the Appendix.

3.7 Forecast Pooling – Empirical Example

However, a significant CW/ENC-NEW still provides useful information when the bootstrapped TU/DMW is insignificant which is what one observes in Table 3.6 (For proof see Appendix). In this section we provide an empirical example of how in such cases one can improve upon the structural model forecast by pooling the structural model forecast and the random walk forecast.

3.7.1 Endogenous vs Exogenous Weights

The question emerges how one can calculate a weight which will produce a forecast with MSFE smaller than the MSFE of the random walk. One can either use endogenous time-varying methods of finding the optimal weight (see Clements and Hendry (1998) pp. 229), or one can impose a fixed weight exogenously. It is conventional wisdom in the literature on forecast pooling that simple averages tend to outperform endogenous weights (See Stock and Watson (2003), Clark and McCracken (2005) and Clements and Hendry (2004)). Clements and Hendry (2004) explain this phenomena with the fact that all endogenous procedures of finding an optimal weight would be biased in the presence of structural breaks (which might be one explanation of the lack of robustness of the models over different forecast windows).⁴⁰ In contrast, having

³⁹Substantial "scale" forecast bias is present in all of the cases where we observe a discrepancy between the TU/DMW and CW/ENC-NEW.

⁴⁰Since endogenous weights are estimated on the basis of data prior to the forecast, a structural break in the recent past or in the near future will lead to biased weights. It is possible that prior to the break, a certain model performs better than the alternative but performs poorly after the structural break. As a result, endogenously determined weights would lead to the forecaster weighting more heavily the model which performed better prior to the break but poorly after it.

a constant weight can serve as an insurance against structural breaks and perform overall better than a time-varying endogenous weight.⁴¹

We test which pooling procedure produces better results – imposing exogenous fixed weights or calculating endogenous weights using the regression method presented in Clements and Hendry (1998) (pp. 229). As expected, our results confirm the conclusion of the literature on forecast pooling that simple means and fixed weights perform better than endogenously calculated optimal weights. As a result, we choose to impose a fixed weight of 0.2 on the structural model forecast and 0.8 on the random walk forecast (which is essentially zero).⁴²

3.7.2 Results After Pooling

Table 3.7.2 presents the "pooled" forecast results where we combine the forecast of the PPP model which incorporates persistent cross-country shocks and the driftless random walk model.

⁴¹Potential structural breaks affect also the degree to which the information provided by the CW and the ENC-NEW is valuable. In the presence of structural breaks, pooling can be appropriate even if the CW and the ENC-NEW are not statistically significant (and of course the bootstrapped TU and DMW are not statistically significant) (see Clements and Hendry (2004)). The reason why this is the case is that the forecaster does not know in advance whether the CW/ENC-NEW will be significant or not if the test statistic is calculated using data from the next regime.

⁴²The results are relatively robust to using a simple average.

Table 3.7.2

"Pooled" Forecast; PPP Specification; One Quarter Ahead; Forecasts Incorporate Common Cross-Country Shocks					
	Theil's U	P-value		DMW	P-value
UK	1.000	0.452		-0.058	0.433
Denmark	0.994	0.050		1.127	0.090
Germany	0.991	0.046		1.355	0.090
Canada	0.962	0.000		2.092	0.005
Japan	0.991	0.071		1.441	0.121
Australia	0.981	0.000		2.151	0.009
Switzerland	0.993	0.123		0.955	0.239
Korea	0.989	0.014		1.645	0.047
Norway	0.997	0.145		0.511	0.189
Sweden	0.991	0.043		1.794	0.044

Forecasts are calculated as 0.2 times the forecast of the Structural PPP Model which incorporates common cross-country shocks

Exploring the results of Table 3.7.2, it is interesting to note that when the forecasts are pooled, the bootstrapped TU becomes statistically significant for the same 7 out of 10 countries for which CW was significant prior to pooling (Table 3.6).⁴³ Therefore, by simply pooling the forecasts (a decision we choose to make after observing the significance of the bootstrapped CW and the ENC-NEW), we are able to outperform the driftless random walk in as many as 7 out of 10 cases. As a result, we would suggest that forecasters interested in finding the forecast which produces the smallest MSFE should not ignore the bootstrapped CW and ENC-NEW test statistics. They should explore the potential of improving their forecast by weighting if the bootstrapped TU and DMW are not statistically significant but the bootstrapped CW and ENC-NEW are significant. However, one should note that while exogenously imposed weights tend to perform relatively well, assigning fixed weights is an ad hoc and sub-optimal process. As a result, it does not guarantee that the same weight will continue performing well after a potential future structural break for example.

⁴³The bootstrapped DMW test statistic is significant in 6 out of 10 cases.

3.7.3 Robustness of the Pooled Forecast with Respect to Different Forecast Windows - Is Pooling Enough?

Finally, we test whether our combined forecasts which incorporate the information presented by the bootstrapped CW and ENC-NEW test statistics are robust over different periods of time. As before, we test the robustness of the results by graphing the bootstrapped TU p-value for different forecast windows. Only those countries for which we have significant results for a number of different forecast windows are included. The "pooled" PPP specification seems to perform exceptionally well in the early-to mid 1980s and relatively well in the early to mid-1990s.

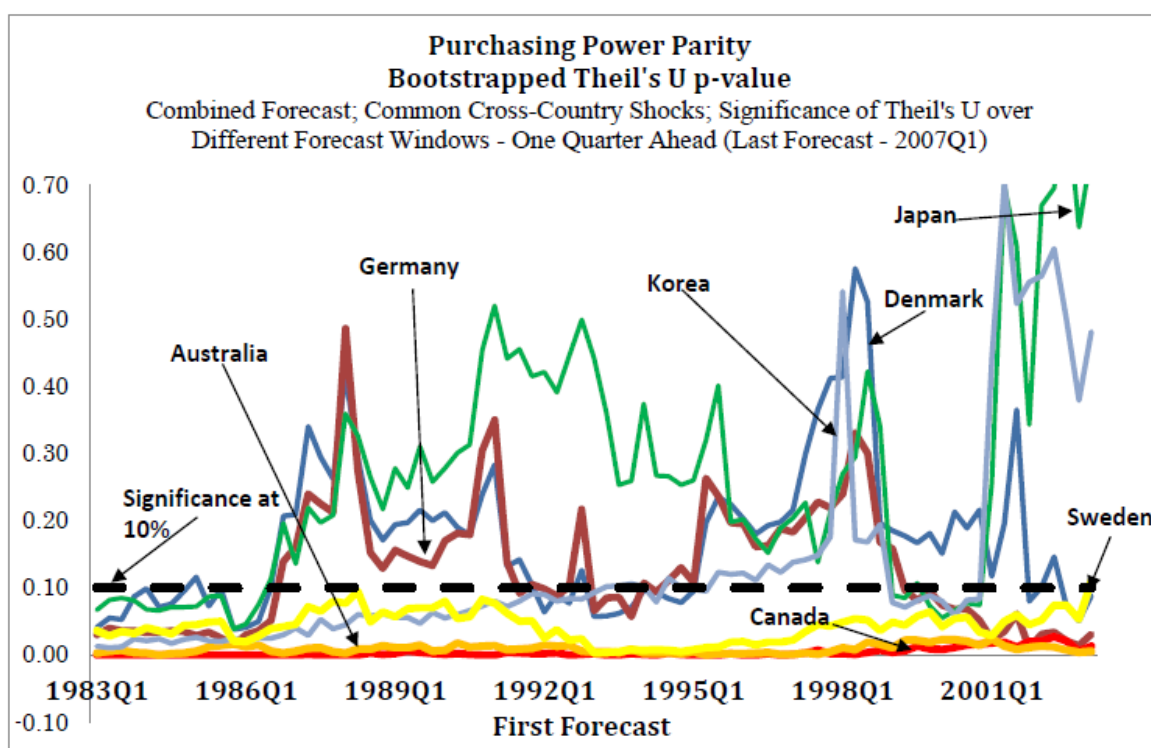


Figure 3.7.3

Nevertheless, the only two countries for which there is robust evidence for forecastability are the commodity exporters Canada and Australia since the bootstrapped TU p-value for these countries is always below 0.1 regardless of the forecast window considered.⁴⁴ It is interesting to note that the forecasting success we observe for Australia and Canada is a result of the way we specify the time dummy effect forecast

⁴⁴There is no guarantee that the relationship for Australia and Canada will be preserved in the future. As a result, one should be careful when interpreting the results. Furthermore, we do not observe similar success when we use the same approach to forecast the exchange rates of other commodity producers such as New Zealand and South Africa. Note also that the results are somewhat robust for Sweden as well.

and not so much of the economic fundamental we use. Even a specification with no structural fundamental (only time and country dummies) produces relatively robust results for Australia and Canada. While these results might be of interest to forecasters, they would most likely be of lesser value to policy makers who are interested in the relationship between structural models and fundamentals.⁴⁵

Overview and Summary of PPP Model (with Common Cross-Country Shocks)

To summarize, panel forecast techniques should improve our ability to forecast exchange rates by increasing the sample size and by allowing for cross-country interactions. We argue that, to an extent, forecasters can exploit cross-country interactions even further via specifying the time dummy effect forecast in a way which captures world economic trends. However, while allowing for the incorporation of cross-country information produces slight improvement over simple panel specifications, it fails to produce robust results for the majority of the countries considered. The only exceptions are the commodity producers – Canada and Australia – but we caution the reader that further investigation of these "success" cases is required. Last but not least, while alternative ways to forecast the time dummy or pooling the structural model coefficient across countries may potentially improve our ability to forecast exchange rates for some countries, one should be cautious when interpreting the results.⁴⁶ If our ability to forecast exchange rates can be attributed solely to "ad hoc" procedures that take into account *unknown* cross-country shocks and common relationships, we still have not improved significantly our knowledge of the relationship between structural models and exchange rates.

3.8 Conclusion

In this paper we attempt to answer the question "Are structural models getting closer to being able to forecast exchange rates at short horizons?" and the answer is "A little." However, over-reliance on asymptotic test statistics in out-of-sample comparisons, misinterpretation of some tests, and failure to sufficiently check robustness to alternative time windows has led many studies to overstate even the relatively thin positive results that have been found. We find that by allowing for common shocks in our panel specification, we

⁴⁵For example, while our results suggest that common cross-country shocks seem to forecast the exchange rates of Australia and Canada relatively well, this result does not help policy makers determine the cause of these shocks or determine the relationship between structural variables and the exchange rate. A recent paper by Chen, Rogoff, and Rossi (2010) is an example of the difficulty of forecasting the exchange rates of commodity producers solely using fundamentals such as commodity prices even when one takes into account structural breaks.

⁴⁶For example, Rapach and Wohar (2004) provide empirical evidence against pooling the monetary model coefficient across countries.

are able to generate some improvement, but even that improvement is not entirely robust to the forecast window, and much of the gain appears to come from non-structural rather than structural factors.

We explore the application of popular new out of sample test statistics such as the Clark and West (2006), Clark and West (2007) and Clark and McCracken (2001) out-of-sample test statistics. We argue that they have been widely misinterpreted as minimum mean square forecast error test statistics and that, in addition, popular simple asymptotic versions may suffer from size distortions. In other words, significant Clark-West and Clark-McCracken test statistics do not always imply that the forecast of the structural model outperforms the forecast of the random walk in terms of mean square forecast error. For this question, statistics such as the bootstrapped Theil's U or Diebold-Mariano/West may be more appropriate (especially given the advances in time series bootstrapping); at the very least, researchers should test the robustness of their results with respect to alternative test statistics.

We note that some researchers may be specifically interested in whether one can reject the null hypothesis that the true model is the random walk model in favor a particular structural model. But we would argue that in the vast majority of applications, policy-makers and practitioners treat the random walk model only as a straw man, and simply want to know whether the structural model can deliver a better forecast and what that forecast is.

We do note that, in principle, a positive CW statistic implies that there does exist some linear combination of the driftless random walk and the structural model that outperforms the naive random walk as measured by relative mean square forecast error. Finding a stable linear combination, however, is tricky and potentially opens up a whole new range of problems. Endogenous methods for finding optimal weights tend to fail due to the presence of structural instability. In practice, fixed exogenous weights tend to perform better, although here too stability is a challenge.

In addition to misinterpretation of the new out-of-sample tests for nested models, some of the excess optimism in the literature can be attributed to the failure to check for robustness over different forecast windows. Regardless of whether one uses new or old structural models, single equation or panel specifications, one of the main problems related to the forecastability of the majority of exchange rates remains - lack of robustness over different time periods. Whether the lack of robustness is due to non-linear functional forms, structural breaks or simply heterogenous market sentiments over time⁴⁷, the literature on exchange rate

⁴⁷One way of explaining the lack of robustness is with the existence of structural breaks which are identified as one of the main problems related to out-of-sample forecasting (see Clements and Hendry (2006), Rossi (2006), Stock and Watson (1996) and Stock and Watson (2003)). Potential model mis-specification could be an alternative explanation. Empirical evidence

forecasting has not been able to develop the tools to produce robust forecasts for the majority of exchange rates. Innovative approaches of overcoming these problems are required in order for the forecasts of structural models to outperform the forecasts of the driftless random walk at short-horizons. Until then, we would call the glass ninety-five percent empty rather than five percent full.

suggest that the relationship between fundamentals and exchange rates can be better represented by non-linear rather than linear functional forms (see Taylor and Peel (2000), Meese and Rose (1991) and Kilian and Taylor (2003)). However, even when forecasters try to account for non-linear functional forms directly (Meese and Rose (1991) and Kilian and Taylor (2003)), or estimate a regime switching model (Marsh (2000) and Daccho and Satchell (1999)), results remain non-robust.

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Appendix A

Supplement to Chapter 1

A.1 Appendix

The functional forms used for the simulations are:

The functional form of the consumer production technology in $t = 1$ is given by

$$F(k_{1s}^T) = \begin{cases} \frac{(1-\gamma)}{\alpha} - \frac{(1-\gamma)e^{-\alpha k^T}}{\alpha} + \gamma k^T > 0 & \text{if } k_{1s}^T > 0 \\ k_{1s}^T & \text{if } k_{1s}^T = 0 \end{cases}$$

where $k_{1s}^T = \min\{0, k_0 - k_{1s}\}$, $0 < \alpha$ is a parameter that controls the concavity of the production technology and $0 < \gamma < 1$ is the refinancing cost. The larger α is, the smaller q_{1l} is, for a given k^T . This functional form guarantees that the assumptions made regarding $F(\cdot)$ are satisfied.

$$q_{1l} = F'(k^T) = (1 - \gamma) e^{-\alpha k^T} + \gamma > 0 \text{ where } 1 > \gamma \geq 0$$

$$F''(k^T) = -\alpha(1 - \gamma) e^{-\alpha k^T} < 0$$

$$F'''(k^T) = \alpha^2(1 - \gamma) e^{-\alpha k^T} > 0$$

$$F(0) = 0; F'(0) = 1; \lim_{k^T \rightarrow \infty} F'(k^T) = \gamma$$

$F(k_{1s}^T)$ and $F'(k_{1s}^T)$ are continuous on $\in [0, \infty)$, $F(k_{1s}^T)$ is at least three times differentiable on $k_{1s}^T \in (0, \infty)$;

$F(0) = 0, F'(0) = 1, F''(0) = F'''(0) = 0, F''(k_{1s}^T) < 0$ on $k_{1s}^T \in (0, \infty)$ and $\lim_{k_{1s}^T \rightarrow \infty} F'(k_{1s}^T) \geq \gamma$. The functional form for the deadweight loss from the bail-out is $\delta(B_l, \chi) = \frac{1}{\chi} B_l^\eta$ where $\eta > 1$.

A.1.1 The Problem of the Consumer

The endowment of the consumer is given by e and the consumer receives it in every period t and state s . I solve the problem of the consumer backwards. In period 2 the consumer maximizes $\max_{k_{2s}^T} (k_{2s}^T - q_{2s}k_{2s}^T + d_{2s} + e)$ taking q_{2s} and the state variable, d_{2s} , as given. The first order condition implies that $q_{2s} = 1$. In period 1, state s , the consumer chooses the amount to invest in the production technology, k_{1s}^T where the price of capital is q_{1s} . He also chooses how much of the period 2 state s asset to buy, d_{2s} , where the price of this asset is p_{2s} and it pays one unit of the consumption good in $t = 2$, state s . Since each consumer is small, he takes all the prices as given. In $t = 1$, state s , the representative consumer maximizes (taking into account the fact that $q_{2s} = 1$)

$$\max_{k_{1s}^T, d_{2s}} [2e + d_{1s} - p_{2s}d_{2s} + d_{2s} + F(k_{1s}^T) - q_{1s}k_{1s}^T]$$

where d_{1s} is a state variable which is the amount the banker promised to pay the consumer in $t = 1$ state s . The first order condition with respect to k_{1s}^T pins down the equilibrium price of capital in $t = 1$ as a function of the amount of "fire-sold" capital, $q_{1s} = F'(k_{1s}^{*T})$. The first order condition with respect to d_{2s} is an Euler equation which implies $p_{2s} = 1$ since the representative consumer is risk neutral and his marginal utility of consumption is 1. Also the inter-temporal discount rate is 1.

After plugging in for the consumer's first order conditions in $t = 1$ and $t = 2$, the $t = 0$ optimization problem of the consumer is given by

$$\max_{d_{1s}} \left[3e - \sum_s \pi_s p_{1s} d_{1s} + \sum_s \pi_s (d_{1s} + F(k_{1s}^{*T}) - q_{1s}k_{1s}^{*T}) \right]$$

The consumer chooses the amount of state contingent asset, d_{1s} , to purchase where he takes the price of the asset $\pi_s p_{1s}$ as given and the asset pays one unit of the consumption good in state s which occurs with probability π_s . The first order condition with respect to d_{1s} implies $p_{1s} = 1$. Notice that since the representative consumer is risk neutral, he will be indifferent how much of the state contingent asset to purchase in period 1 and 2.

A.1.2 N Banks – No Commitment

In this section of the Appendix, I solve the optimization problem of banker i where the policy maker provides an optimal bail-out in $t = 1$. Also the policy maker has an access to two ex-ante (period zero) policy instruments. The first ex-ante policy instrument is a minimum bank capital requirement defined as $\rho^i k_0^i \leq n$ where ρ^i is the minimum capital ratio. The second ex-ante policy instrument is a limit on the payment banker i pledges in the low state to the consumer, ν^i , (i.e. a limit on the amount of put options banker i can sell) $d_{1l}^i \leq \nu^i$. In order to remain close to reality, I assume that no commitment instruments are available to the policy maker. As a result, the optimal targeted bail-out, B_l^i , will be determined in $t = 1$ and banker i will internalize the fact that his actions in period $t = 0$ will affect B_l^i . In contrast, ρ^i and ν^i are predetermined in the beginning of period $t = 0$, before banker i makes any decisions. Also I assume that banker i takes into account the first order conditions of the consumers which pin down prices $p_{1s} = p_{2s} = 0$. $q_0 = q_{2s} = 1$ and $q_{1s} = F'(k_{1s}^T)$.

The actions in reverse order are the following. In $t = 2$, all bankers produce and pay out all the profits as dividends to the consumers. *At the end of $t = 1$* , banker i maximizes the dividend payment in the last period by choosing $\{k_{1s}^i, d_{2s}^i\}$ and taking as given the state variables $\{B_l^i, k_0^i, d_{1s}^i\}$ and the policy instruments predetermined in $t = 0$, $\{\rho^i, \nu^i\}$. I also solve out for the price vector using the first order conditions of the consumer. At the end of $t = 1$, banker i maximizes

$$\max_{k_{1s}^i, d_{2s}^i} (A + 1 - \gamma) k_{1s}^i - d_{2s}^i$$

subject to the collateral constraint in $t = 1$

$$d_{2s}^i \leq \theta (1 - \gamma) k_{1s}^i \quad [\lambda_{2s}^i]$$

where the Lagrangians are given in square brackets. Banker i also takes into account the period one budget constraint

$$k_{1s}^i F'(k_{1s}^T) + d_{1s}^i \leq (F'(k_{1s}^T) + a_{1s} - \gamma) k_0^i + B_s^i + d_{2s}^i \quad [z_{1s}^{i,1}]$$

From now on till the rest of this subsection I set $B_h^i = 0$ since I assumed bail-outs are prohibitively costly if there is no fire sale (no crisis) and in Proposition 1.3.1 I prove that given Assumptions 1.1-1.6, $q_{1h} = 1$ and $q_{1l} < 1$ and, hence, there is no fire sale in the crisis state.¹ From the first order conditions and the

¹The implicit assumption is that there is a large fixed cost of providing a transfer of money from the consumers to bankers if there is no fire sale (no crisis). This assumption can be relaxed and all the results remain.

constraints one can solve for $\{k_{1s}^i, d_{2s}^i\}$ as a function of the state variables and prices.

First order condition with respect to k_{1s}^i :

$$z_{1s}^{i,1} = \frac{A + 1 - \gamma + \lambda_{2s}^i \theta (1 - \gamma)}{\left(F' (k_{1s}^T) + \frac{1}{N} F'' (k_{1s}^T) k_{1s}^{i,T} \right)} > 1$$

The fact that $z_{1s}^{i,1} > 1$ comes from the assumptions that $F' (k_{1s}^T) \leq 1$, $\frac{1}{N} F'' (k_{1s}^T) k_{1s}^{i,T} < 0$ and $A - \gamma > 0$ and from the fact that $\lambda_{2s}^i \geq 0$.

First order condition with respect to d_{2s}^i :

$$-1 - \lambda_{2s}^i + z_{1s}^{i,1} = 0$$

Next I prove that $\lambda_{2s}^i > 0$. Since $z_{1s}^{i,1} > 1$, then $\lambda_{2s}^i = z_{1s}^{i,1} - 1 > 0$ which implies that the period one collateral constraint always binds and $d_{2s}^i = \theta (1 - \gamma) k_{1s}^i$.

$$z_{1s}^{i,1} = \frac{A + (1 - \theta) (1 - \gamma)}{F' (k_{1s}^T) + \frac{1}{N} F'' (k_{1s}^T) k_{1s}^{i,T} - \theta (1 - \gamma)}$$

At the beginning of $t = 1$, first, banker i repays the promised debt d_{1s}^i to the consumers. After that, if the low state is realized, the policy maker chooses B_l^i given the state variables $\{k_0^i, d_{1s}^i\}$ and the policy instruments predetermined in $t = 0$, $\{\rho^i, \nu^i\}$. He also takes into account that the first order conditions of banker i at the end of $t = 1$ which are a function of B_l^i . Assuming that the Central Planner places equal weight on all consumers who are risk neutral, the objective function of the policy maker in $t = 1$ in the low state is to maximize period 2 total output (i.e. the consumption of the consumers who also own the banks). The optimization problem of the policy maker in the beginning of $t = 1$ in the low state is

$$\begin{aligned} & \max_{k_{1l}^i, B_l^i} 2e + F (k_{1l}^T) - F' (k_{1l}^T) k_{1l}^T - \delta (B_l) + d_{1l} - p_{2l} d_{2l} + d_{2l} \\ & + \sum_{i=1}^N \frac{1}{N} \left[(A + 1 - \gamma) k_{1l}^i - d_{2l}^i - B_l^i + \right. \\ & \left. z_{1l}^{i,1,P} ((F' (k_{1l}^T) + a_{1l} - \gamma) k_0^i + B_l^i + d_{2s}^i - k_{1l}^i F' (k_{1l}^T) - d_{1l}^i) \right] \end{aligned}$$

where $z_{1l}^{i,1,P}$ is the Lagrangian on the period one budget constraint in the low state which pins down k_{1l}^i chosen by banker i at the end of $t = 1$. $(A + 1 - \gamma) k_{1l}^i - d_{2l}^i$ are the dividends paid by banker i to the equity owners (consumers) in $t = 2$ in the low state, $F (k_{1l}^T) - F' (k_{1l}^T) k_{1l}^T$ are the profits of the consumers from operating their production technology if a fire sale is present. $d_{1l} + d_{2l}$ are the period one and two payments

by all bankers to the consumers and $p_{2l}d_{2l}$ is the amount lent to the bankers in $t = 1$ by the consumers. $\delta(B_l) + B_l$ is the cost of the bail out – the direct cost plus the deadweight loss from taxing). I assume that the endowment of the consumer, e , is large enough so that $B_l < e + d_{1l} - p_{2l}d_{2l}$.

Taking into account the fact that $\lambda_{2s}^i > 0$, and that $p_{2s} = 1$, the optimization problem can be re-written as

$$\begin{aligned} & \max_{k_{1l}^i, B_l^i} 2e + F(k_{1l}^T) - F'(k_{1l}^T) k_{1l}^T - \delta(B_l) + d_{1l} \\ & + \sum_{i=1}^N \frac{1}{N} \left[\begin{aligned} & (A + (1 - \theta)(1 - \gamma)) k_{1s}^i - B_l^i + \\ & z_{1l}^{i,1,P} ((F'(k_{1l}^T) + a_{1l} - \gamma) k_{1l}^i + B_l^i + \theta(1 - \gamma) k_{1s}^i - k_{1l}^i F'(k_{1l}^T) - d_{1l}^i) \end{aligned} \right] \end{aligned}$$

First order condition with respect to k_{1l}^i :

$$(F''(k_{1l}^T) k_{1l}^T + A + (1 - \theta)(1 - \gamma)) + z_{1l}^{i,1,P} (\theta(1 - \gamma) - F'(k_{1l}^T)) = F''(k_{1l}^T) \sum_{j=1}^N \frac{1}{N} z_{1l}^{j,1,P} k_{1l}^{j,T} \quad (\text{A.1.1})$$

Since equation A.1.1 holds for every i , $z_{1l}^{i,1,P}$ is the same for every i and one can simplify equation A.1.1 as

$$z_{1l}^{i,1,P} = z_{1l}^{1,P} = \frac{F''(k_{1l}^T) k_{1l}^T + A + (1 - \theta)(1 - \gamma)}{F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma)} > 1 \quad (\text{A.1.2})$$

where $z_{1l}^{1,P} > 1$ come from the assumptions $F'(k_{1s}^T) \leq 1$, $\frac{1}{N} F''(k_{1s}^T) k_{1s}^{i,T} < 0$ and $A - \gamma > 0$.

The first order condition with respect to B_l^i is

$$1 + \delta'(B_l) = z_{1l}^{1,P} (k_{1l}^T) \quad (\text{A.1.3})$$

The policy maker is indifferent whom to give the bail-out to. He wants to control the size of the fire sale given the cost of the marginal bail-out but he is indifferent whether he gives all the money to a single bank or splits it equally among all banks. I will assume a symmetric equilibrium in order to solve the model, i.e. $B_l^i = B_l$.

At the end of period $t = 0$, banker i chooses $\{k_{1s}^i, d_{1s}^i\}$, taking as given $\{\rho^i, \nu^i\}$ and internalizing his effect on B_l^i and on his own future actions. I plug in for d_{2s}^i and take into account that k_{1s}^i is pinned down by the period one budget constraint while B_l^i is pinned down by the first order condition of the policy maker

with respect to B_l^i , equation A.1.3, in the beginning of period $t = 1$. At the end of period $t = 0$, banker i optimizes the expected value of period two dividends given by

$$\max_{k_0^i, d_{1s}^i, k_{1s}^i} \sum_{s \in \{l, h\}} \pi_s (A + (1 - \theta)(1 - \gamma)) k_{1s}^i$$

subject to the period one budget constraint

$$k_{1s}^i (F' (k_{1s}^T) - \theta(1 - \gamma)) + d_{1s}^i \leq (F' (k_{1s}^T) + a_{1s} - \gamma) k_0^i + B_s^i \quad \left[\pi_s z_{1s}^{i,0} \right] \quad (\text{A.1.4})$$

the period zero budget constraint

$$k_0^i - n \leq \sum_s \pi_s d_{1s}^i \quad \left[z_0^i \right]$$

the period zero borrowing constraint

$$d_{1s}^i \leq \theta (F' (k_{1s}^T) - \gamma) k_0^i \quad \left[\pi_s \lambda_{1s}^i \right]$$

the minimum bank capital requirement

$$\rho^i k_0^i \leq n \quad \left[\iota^i \right]$$

and finally subject to the limit on put options sold by banker i (i.e. the promised payment in the crisis state by banker i)

$$d_{1l}^i \leq \nu^i \quad \left[\pi_l \varphi^i \right]$$

First order condition with respect to k_0^i :

$$\begin{aligned} & \sum_s \pi_s z_{1s}^{i,0} \left(F' (k_{1s}^T) + a_{1s} - \gamma + \frac{1}{N} F'' (k_{1s}^T) k_{1s}^{i,T} + \frac{\partial B_s^i}{\partial k_0^i} \right) \\ & - z_0^i - \rho^i \iota^i + \sum_s \pi_s \lambda_{1s}^i \theta \left(F' (k_{1s}^T) - \gamma + \frac{1}{N} F'' (k_{1s}^T) k_0^i \right) = 0 \end{aligned} \quad (\text{A.1.5})$$

First order condition with respect to k_{1s}^i :

$$A + (1 - \theta)(1 - \gamma) + z_{1s}^{i,0} \left[-\frac{1}{N} F''(k_{1s}^T) k_{1s}^{i,T} + \frac{\partial B_s^i}{\partial k_{1s}^i} - (F'(k_{1s}^T) - \theta(1 - \gamma)) \right] + \lambda_{1s}^i \theta \left[-\frac{1}{N} F''(k_{1s}^T) k_0^i \right] = 0 \quad (\text{A.1.6})$$

First order condition with respect to d_{1l}^i :

$$-z_{1l}^{i,0} + z_0^i - \lambda_{1l}^i - \varphi^i = 0$$

First order condition with respect to d_{1h}^i :

$$-z_{1h}^{i,0} + z_0^i - \lambda_{1h}^i = 0$$

where using equation A.1.3 one can show that $\frac{\partial B_l^i}{\partial k_0^i} = \frac{1}{\delta''(B_l^i)_N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} \cdot \frac{\partial B_l^i}{\partial k_{1l}^T} = -\frac{1}{\delta''(B_l^i)_N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T}$.

A.1.3 Constrained Central Planner's Problem – No Commitment

Since I assume no commitment, I solve the problem backwards. In period one the Central Planner maximizes the welfare of the consumers (who are also the bank owners), which in this environment with risk neutral consumers, coincides with maximizing aggregate output.

$$\max_{k_{1s}, B_s, d_{2s}} 2e - (\delta(B_s) + B_s) + F(k_{1s}^T) - F'(k_{1s}^T) k_{1s}^T + d_{1s} + (A + 1 - \gamma) k_{1s} - d_{2s}$$

The optimization problem is subject to the collateral constraint in $t = 2$

$$d_{2s} \leq \theta(1 - \gamma) k_{1s} \quad \left[\lambda_{2s}^{CP} \right].$$

The optimization problem in $t = 1$ is also subject to the period one budget constraint

$$k_{1s} F'(k_{1s}^T) + d_{1s} \leq (F'(k_{1s}^T) + a_{1s} - \gamma) k_0 + B_s + d_{2s} \quad \left[z_{1s}^{1,CP} \right]$$

From the first order condition with respect to k_{1s}

$$z_{1s}^{1,CP} = \frac{F''(k_{1s}^T) k_{1s}^T + A + 1 - \gamma + \lambda_{2s}^{CP} \theta(1 - \gamma)}{F'(k_{1s}^T) + F''(k_{1s}^T) k_{1s}^T}$$

The first order condition with respect to d_{2s} is

$$z_{1s}^{1,CP} = 1 + \lambda_{2s}^{1,CP} \geq 1 \quad (\text{A.1.7})$$

First order condition with respect to B_s

$$z_{1s}^{1,CP} = 1 + \delta' (B_s)$$

First I prove that $\lambda_{2s}^{1,CP} > 0$. Since $q_{1s} \leq 1$ and $A + 1 - \gamma + \lambda_{2s}^{1,CP} \theta (1 - \gamma) > 1$ then $z_{1s}^{1,CP} > 1$. From equation A.1.7 $\lambda_{2s}^{1,CP} = z_{1s}^{1,CP} - 1 > 0$ which completes the proof that $\lambda_{2s}^{1,CP} > 0$. Hence, $d_{2s} = \theta (1 - \gamma) k_{1s}$. Re-writing the first order condition with respect to k_{1s} using the fact that $\lambda_{2s}^{1,CP} = z_{1s}^{1,CP} - 1$

$$z_{1s}^{1,CP} = \frac{F'' (k_{1s}^T) k_{1s}^T + A + (1 - \gamma) (1 - \theta)}{F' (k_{1s}^T) + F'' (k_{1s}^T) k_{1s}^T - \theta (1 - \gamma)} \quad (\text{A.1.8})$$

The Central Planner's optimization problem in $t = 0$ becomes (taking into account that in equilibrium $p_{2s} = 1$ and $p_{1s} = 1$)

$$\max_{k_0, \{k_{1s}, d_{1s}\}_{s=l, h}} 3e - \sum \pi_{1s} d_{1s} + \sum \pi_s \left[\begin{array}{c} d_{1s} - d_{2s} + d_{2s} + F (k_{1s}^T) \\ -F' (k_{1s}^T) k_{1s}^T - B_s - \delta (B_s) + (A + 1 - \gamma) k_{1s} - d_{2s} \end{array} \right]$$

Using the fact that $d_{2s} = \theta (1 - \gamma) k_{1s}$

$$\max_{k_0, \{k_{1s}, d_{1s}\}_{s=l, h}} 3e + \sum \pi_s [F (k_{1s}^T) - F' (k_{1s}^T) k_{1s}^T - B_s - \delta (B_s) + (A + (1 - \gamma) (1 - \theta)) k_{1s}]$$

The optimization problem is subject to the following constraints. The budget constraint in $t = 1$

$$k_{1s} (F' (k_{1s}^T) - \theta (1 - \gamma)) + d_{1s} \leq (F' (k_{1s}^T) + a_{1s} - \gamma) k_0 + B_s \quad \left[\pi_s z_{1s}^{0,CP} \right]$$

the budget constraint in period zero

$$k_0 \leq n + \sum_s \pi_s d_{1s} \quad [z_0^{CP}]$$

and the period one collateral constraint

$$d_{1s} \leq \theta (F' (k_{1s}^T) - \gamma) k_0 \quad \left[\pi_s \lambda_{1s}^{CP} \right] \quad (\text{A.1.9})$$

First order condition with respect to k_0 :

$$\sum \pi_s \left(\begin{array}{c} -F''(k_{1s}^T) k_{1s}^T - \frac{\partial B_s}{\partial k_0} (1 + \delta'(B_s)) + \\ z_{1s}^{0,CP} \left(F'(k_{1s}^T) + a_{1s} - \gamma + F''(k_{1s}^T) k_{1s}^T + \frac{\partial B_s}{\partial k_0} \right) \\ + \lambda_{1s}^{CP} \theta [F'(k_{1s}^T) - \gamma + F''(k_{1s}^T) k_0] \end{array} \right) = z_0^{CP} \quad (\text{A.1.10})$$

where $\frac{\partial B_s}{\partial k_0} = \frac{1}{\delta''(B_s)} \frac{\partial z_{1s}^{1,CP}}{\partial k_{1s}^T}$. First order condition with respect to k_{1s} :

$$\begin{aligned} & F''(k_{1s}^T) k_{1s}^T - \frac{\partial B_s}{\partial k_{1s}} (1 + \delta'(B_s)) + A + (1 - \gamma)(1 - \theta) \\ & + z_{1s}^{0,CP} \left[-F''(k_{1s}^T) k_{1s}^T + \frac{\partial B_s}{\partial k_{1s}} - (F'(k_{1s}^T) - \theta(1 - \gamma)) \right] - \lambda_{1s}^{CP} \theta \tilde{F}''(k_{1s}^T) k_0 = 0 \end{aligned} \quad (\text{A.1.11a})$$

where $\frac{\partial B_s}{\partial k_{1s}} = -\frac{1}{\delta''(B_s)} \frac{\partial z_{1s}^{1,CP}}{\partial k_{1s}^T}$. First order condition with respect to d_{1s} :

$$z_0^{CP} - z_{1s}^{0,CP} - \lambda_{1s}^{CP} = 0$$

A.1.4 Proofs

Proposition 1.3.1

Proposition 1.3.1: *Given Assumptions 1.1-1.7 and the Assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, considering a symmetric equilibrium with an ex-post optimal bail-out and no ex-ante regulation:*
(i) There is no fire sale in the high state, $q_{1h} = 1$ and there is a fire sale in the low state, $q_{1l} < 1$. (ii) Given the additional Assumption 1.8 (required only for the $N < \infty$ case),

Assumption 1.8

$$\left(\frac{1}{N} + 1 \right) F''(k_{1l}^T) - \frac{F''(k_{1l}^T) \left(1 - 2z_{1l}^{1,P}(k_{1l}^T) \right)}{\left(F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma) \right)^2} \frac{F''(k_{1l}^T)}{\delta''(B_l) N} \left[4 + \frac{\delta'''(B_l)}{(\delta''(B_l))^2} \left(1 - 2z_{1l}^{1,P} \right) \right] < 0$$

and $F'''(k_{1l}^T) = 0$, the equilibrium is unique and exists and is one of the following types: Type 1) $z_0^* = z_{1l}^* > z_{1h}^*$; ($\lambda_{1l}^* = 0, \lambda_{1h}^* > 0$) (interior equilibrium) Type 2) $z_0^* > z_{1s}^*$; ($\lambda_{1s}^* = 0$) (corner equilibrium where

the banker borrows to the maximum in $t = 0$) where

$$z_0^* = \sum_s \pi_s \left[z_{1s}^* \left(F'(k_{1s}^T) + a_{1s} - \gamma + \frac{1}{N} F''(k_{1s}^T) k_{1s}^T + \frac{1}{\delta''(B_s)N} \frac{\partial z_{1s}^{1,P}}{\partial k_{1s}^T} \right) + \lambda_{1s}^* \theta \left(F'(k_{1s}^T) - \gamma + \frac{1}{N} F''(k_{1s}^T) k_0 \right) \right] \quad (\text{A.1.12})$$

$$z_{1s}^* = \frac{A + (1 - \theta)(1 - \gamma) - \lambda_{1s}^* \theta \frac{1}{N} F''(k_{1s}^T) k_0}{\frac{1}{N} F''(k_{1s}^T) k_{1s}^T + \frac{1}{\delta''(B_s)N} \frac{\partial z_{1s}^{1,P}}{\partial k_{1s}^T} + F'(k_{1s}^T) - \theta(1 - \gamma)} \quad (\text{A.1.13})$$

and $z_{1s}^* = z_{1s}^{0,*}$. The optimal bail-out is pinned down by

$$1 + \delta'(B_l) = z_{1l}^{1,P} = \frac{F''(k_{1l}^T) k_{1l}^T + A + (1 - \theta)(1 - \gamma)}{F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma)} \quad (\text{A.1.14})$$

where $\frac{\partial B_s^i}{\partial k_0^i} = -\frac{\partial B_s^i}{\partial k_{1s}^i} = \frac{1}{\delta''(B_s)N} \frac{\partial z_{1s}^{1,P}}{\partial k_{1s}^T}$, and the first order conditions with respect to d_{2s} and d_{1s} imply $\lambda_{2s}^* > 0$, $\lambda_{1s}^* = z_0^* - z_{1s}^* \geq 0$.

Before I prove Proposition 1.3.1 I prove Lemmas **A.1.1** and **A.1.2**.

Lemma A.1.1 *Conditional on Assumptions 1.1-1.6 and conditional on a fire sale in the low state, $k_{1l}^T > 0$, then $\frac{\partial B_l(k_{1l}^T)}{\partial k_{1l}^T} > 0$ and $\frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} > 0$.*

Proof of Lemma A.1.1. From the first order condition of the policy maker in the beginning of $t = 1$ given by equations A.1.2 and A.1.3,

$$1 + \delta'(B_l) = z_{1l}^{1,P}(k_{1l}^T) = \frac{F''(k_{1l}^T) k_{1l}^T + A + (1 - \theta)(1 - \gamma)}{F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma)} > 1$$

Since $z_{1l}^{1,P} > 1$ (see Section A.1.2.), $F''(k_{1l}^T) < 0$ and also from Assumption 1.4

$$\frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} = \frac{(F''(k_{1l}^T) + F'''(k_{1l}^T) k_{1l}^T) (1 - z_{1l}^{1,P}(k_{1l}^T)) - z_{1l}^{1,P}(k_{1l}^T) F''(k_{1l}^T)}{F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma)} > 0 \quad (\text{A.1.15})$$

Since $\frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} > 0$ and the deadweight loss function from the bail-out is convex with respect to B_l , $\delta''(B_l) > 0$, one can show that larger fire sale leads to a larger optimal bail-out

$$\frac{\partial B_l(k_{1l}^T)}{\partial k_{1l}^T} = \frac{1}{\delta''(B_l)} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} > 0$$

Lemma A.1.2 *Given Assumptions 1.1-1.7 and considering a symmetric equilibrium, there is never a fire sale in the high state, $q_{1h} = 1$ and there is a fire sale in the low state, $q_{1l} < 1$.*

Proof of Lemma A.1.2. First, I show that assuming a symmetric equilibrium, then $q_{1h} = 1$. In Section A.1.2., I proved that the period one borrowing constraint is binding in both states of nature $\lambda_{2s} > 0$. Therefore, taking into account that $d_{2s} = \theta(1 - \gamma)k_{1s}$, I can re-write the budget constraint in $t = 1$ and state s as

$$(k_{1s} - k_0)(q_{1s} - \theta(1 - \gamma)) \leq (a_{1s} - \gamma + \theta(1 - \gamma))k_0 + B_s - d_{1s}$$

Also using the fact that the maximum promised payment in $t = 1$ in the high state is pinned down by the binding borrowing constraint, $d_{1h} = \theta(q_{1h} - \gamma)k_0$ and also from Assumption 1.5, $a_{1h} > \gamma$ then

$$(k_{1h} - k_0)(q_{1h} - \theta(1 - \gamma)) = (a_{1h} - \gamma + \theta(1 - \gamma))k_0 + B_h - d_{1h} \geq (a_{1h} - \gamma + \theta(1 - q_{1h}))k_0 + B_h > 0$$

Since $k_{1h} - k_0 > 0$, there is no fire sale in the high state, $q_{1h} = 1$.

The proof that $q_{1l} < 1$ proceeds in two steps. **Step 1)** First, I show that, given Assumption 1.2, if there is no fire sale in $t = 1$, in the low state, $q_{1l} = 1$, the only possible equilibrium is the corner one where the bankers borrow to the maximum in $t = 0$ (Type 2 equilibrium). This implies that if the equilibrium is not the corner equilibrium of Type 2, then there must be a fire sale in the low state. Also since $k_0 = 0$ implies no fire sale in equilibrium (no borrowing in period zero and only lending to the maximum), proving the first step automatically implies that the corner solution $k_0 = 0$ is impossible as well justifying why I ignored the $k_0 \geq 0$ constraint when solving for the banker's problem. **Step 2)** The second step is to show that given Assumption 1.6, even if the Type 2 equilibrium is the optimal one, there will be always a fire sale in $t = 1$ in the low state. Steps one and two are sufficient to prove that there is always a fire sale in equilibrium in the crisis state given the assumptions made. Finally, via simulations I prove that the set of parameters for which the Type 1 equilibrium is the optimal one is non-empty. Assumption 1.6 is the most general assumption which guarantees the presence of a fire sale in the crisis state.

Step 1) I will prove that conditional on Assumption 1.2 being satisfied, given that I proved that $q_{1h} = 1$, and if I assume that $q_{1l} = 1$, then $z_0^* > z_{1s}^*$. This will imply that the only possible equilibrium if there are no fire sales is of Type 2 (the corner equilibrium). Since $q_{1s} = 1$, then $\frac{\partial B_s^i}{\partial k_{1s}^i} = 0$ and $F''(k_{1s}^T) = 0$, and one can re-write the first order condition with respect to k_0 as $\sum_s \pi_s [z_{1s}^*(1 + a_{1s} - \gamma) + \lambda_{1s}\theta(1 - \gamma)] = z_0^*$. Let's consider all the possible cases based on all the possible combinations of λ_{1h} and λ_{1l} . Case 1) $\lambda_{1s} = 0$. If $\lambda_{1s} = 0$, then $z_{1s}^* = z_0^*$ which is impossible since if Assumption 1.2 was satisfied then $z_{1s}^* \sum_s \pi_s (1 + a_{1s} - \gamma) = z_0^* > z_{1s}^*$. Case 2) $\lambda_{1l} = 0$ and $\lambda_{1h} > 0$. This case is impossible since it implies that $z_0^* = z_{1l}^* > z_{1h}^* = z_0^* - \lambda_{1h}$. However, from the first order condition with respect to k_{1s} , $z_{1l}^* = z_{1h}^*$ which is a contradiction. Case 3) $\lambda_{1h} = 0$

and $\lambda_{1l} > 0$. Similarly, the case $\lambda_{1h} = 0$ and $\lambda_{1l} > 0$ is impossible due to the same argument as to why Case 3 is impossible. Case 4) $\lambda_{1s} > 0$. This is the case where banker i borrows to the maximum in $t = 0$ (Type 2 equilibrium). From the first order condition with respect to d_{1s} , $z_{1s}^* + \lambda_{1s}^* = z_0^* > z_{1s}^*$. One can re-write z_0^* as

$$z_0^* = z_{1s}^* \frac{\sum_s \pi_s [(1-\gamma)(1-\theta) + a_{1s}]}{(1-\theta(1-\gamma))} \quad (\text{A.1.16})$$

$z_0^* > z_{1s}^*$ implies

$$\frac{\sum_s \pi_s [(1-\gamma)(1-\theta) + a_{1s}]}{(1-\theta(1-\gamma))} > 1 \quad (\text{A.1.17})$$

One can show that the condition $z_0^* > z_{1s}^*$ is satisfied as long as Assumption 1.2 is satisfied which completes the proof that as long as Assumption 1.2 is satisfied and there is no fire sale, the banker always optimally borrows to the maximum in $t = 0$ (the equilibrium is of Type 2).

Step 2) Next, I prove that given Assumption 1.6 and if the banker borrows to the maximum in $t = 0$ (the equilibrium is of Type 2), there is always a fire sale in $t = 1$ in the low state. To do that, I show that if the banker borrows to the maximum, there exists a unique $k_{1l}^{T,\max}$ and $k_{1l}^{T,\max} > 0$.

To solve for k_0^{\max} as a function of $k_{1l}^{T,\max}$ I use the period zero budget constraint $k_0^{\max} = \sum_s \pi_s \theta (q_{1s} - \gamma) k_0^{\max} + n$ (imposing the condition that the period zero borrowing constraints are binding).

$$k_0^{\max} = \frac{n}{1 - \pi_h \theta (1-\gamma) - \pi_l \theta (F'(k_{1l}^T) - \gamma)} \quad (\text{A.1.18})$$

First, I prove by contradiction that if k_{1l}^T exists, then $k_{1l}^T > 0$. Assume that $k_{1l}^T = 0$, and banker i borrows to the maximum in $t = 0$. Re-writing the budget constraint in the low state in $t = 1$

$$\begin{aligned} k_{1l} (1 - \theta (1 - \gamma)) &= \frac{(a_{1l} + (1 - \gamma) (1 - \theta)) n}{1 - \theta (1 - \gamma)} + (\delta')^{-1} \left(\frac{A - \gamma}{1 - \theta (1 - \gamma)} \right) \\ &= \frac{(a_{1l} - \gamma) n}{1 - \theta (1 - \gamma)} + (\gamma + (1 - \gamma) (1 - \theta)) k_0^{\max} + (\delta')^{-1} \left(\frac{A - \gamma}{1 - \theta (1 - \gamma)} \right) \end{aligned}$$

Given Assumption 1.6,

$$(k_{1l} - k_0^{\max}) (1 - \theta (1 - \gamma)) = \frac{(a_{1l} - \gamma) n}{1 - \theta (1 - \gamma)} + (\delta')^{-1} \left(\frac{A - \gamma}{1 - \theta (1 - \gamma)} \right) < 0$$

which implies that $k_{1l} - k_0^{\max} = -k_{1l}^T < 0$. This contradicts the assumption that $k_{1l}^T = 0$. Therefore, if k_{1l}^T

exists and the banker borrows to the maximum in $t = 0$, then $k_{1l}^T > 0$.

Next I prove existence and uniqueness of $k_{1l}^{T,\max} = k_{1l}^T(k_0^{\max})$. Due to Part 1) I can focus only on the range $k_{1l}^T \in (0, k_0^{\max}]$. Using the fact that $d_{1s} = \theta(1 - \gamma)k_{1s}^{\max}$ and $d_{2s} = \theta(q_{1s} - \gamma)k_0^{\max}$ and from the period one budget constraint in the low state

$$H(k_{1l}^T; k_0 = k_0^{\max}) = (a_{1l} - \gamma + \theta(1 - \gamma) - \theta(F'(k_{1l}^T) - \gamma))k_0^{\max} + B_l(k_{1l}^T) + (F'(k_{1l}^T) - \theta(1 - \gamma))k_{1l}^T \quad (\text{A.1.19})$$

Next, I prove that $H(k_{1l}^{T,\max} = k_0^{\max}; k_0 = k_0^{\max}) > 0$, $H(k_{1l}^T = 0; k_0 = k_0^{\max}) < 0$ and $\frac{\partial H(k_{1l}^T)}{\partial k_{1l}^T} > 0$. This guarantees that there exists a unique $k_{1l}^{T,\max} > 0$.

$$H(k_{1l}^T = k_0^{\max}; k_0 = k_0^{\max}) = (a_{1l} + (F'(k_0^{\max}) - \gamma)(1 - \theta))k_0^{\max} + B_l(k_0^{\max}) > 0$$

Given Assumption 1.6

$$H(k_{1l}^T = 0; k_0 = k_0^{\max}) = \frac{(a_{1l} - \gamma)n}{1 - \theta(1 - \gamma)} + (\delta')^{-1} \left(\frac{A - \gamma}{1 - \theta(1 - \gamma)} \right) < 0 \quad (\text{A.1.20})$$

Since $F''(k_{1l}^T) < 0$, $B'_l(k_{1l}^T) > 0$ (derived in Lemma A.1.1), from Assumption 1.4, and since Assumption 1.3 implies $\theta(1 - F'(k_{1l}^T)) < \theta(1 - \gamma) < \gamma - a_{1l}$, then one can show that

$$\begin{aligned} \frac{\partial H(k_{1l}^T)}{\partial k_{1l}^T} &= -\theta F''(k_{1l}^T)k_0^{\max} + (a_{1l} - \gamma + \theta(1 - F'(k_{1l}^T))) \frac{\partial k_0^{\max}(k_{1l}^T)}{\partial k_{1l}^T} \\ &\quad + B'_l(k_{1l}^T) + F'(k_{1l}^T) - \theta(1 - \gamma) + F''(k_{1l}^T)k_{1l}^T > 0 \end{aligned}$$

where after differentiating equation A.1.18

$$\frac{\partial k_0^{\max}(k_{1l}^T)}{\partial k_{1l}^T} = \frac{\pi_l \theta F''(k_{1l}^T)k_0^{\max}}{1 - \pi_h \theta(1 - \gamma) - \pi_l \theta(F'(k_{1l}^T) - \gamma)} < 0 \text{ if } k_{1l}^T > 0$$

The fire sale that will emerge in equilibrium if the bank borrows to the maximum in $t = 0$ is pinned down by $H(k_{1l}^T; k_0 = k_0^{\max}) = 0$. This completes the proof that given Assumptions 1.1-1.6 there exists a unique equilibrium $k_{1l}^{T,\max}$ and $k_{1l}^{T,\max} > 0$ (i.e. $q_{1l}(k_0^{\max}) < 1$).

Proof of Proposition 1.3.1.

PART 1) First, I prove that the only two types of equilibria that can emerge are of Type 1 and Type 2.

In order to characterize the equilibrium, I consider all four possible combinations of whether the Lagrangians on the period one borrowing constraint, λ_{1h} and λ_{1l} , are greater than or equal to zero (given that I already proved that the period 2 borrowing constraints are always binding $\lambda_{2s} > 0$). First let's consider the case $\lambda_{1s}^* = 0$ ($z_{1h}^* = z_0^* = z_{1l}^*$) and prove that given Assumption 1.2 this case will never be an equilibrium. If the policy maker does not have an access to ex-ante regulatory instruments, one can re-write the first order conditions of the banker with respect to k_{1s} and k_0 as

From the first order condition with respect to k_0

$$z_{1l}^* = \frac{A + (1 - \theta)(1 - \gamma)}{\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - \theta(1 - \gamma)} = \frac{A + (1 - \theta)(1 - \gamma)}{1 - \theta(1 - \gamma)} = z_{1h}^* \quad (\text{A.1.21})$$

$$1 - F'(k_{1l}^T) = \frac{1}{N} \left[F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} \right]$$

From the first order condition with respect to k_{1s}

$$A + (1 - \theta)(1 - \gamma) + z_0^* \sum_s \pi_s (\theta(1 - \gamma) + a_{1s} - \gamma) = z_0^* \quad (\text{A.1.22})$$

Plugging $z_{1h}^* = z_{1l}^* = z_0^* = \frac{A + (1 - \theta)(1 - \gamma)}{1 - \theta(1 - \gamma)}$ in equation A.1.22 and simplifying implies $1 = \sum_s \pi_s (1 + a_{1s} - \gamma)$.

However, given Assumption 1.2, the equation above will not be satisfied and hence $\lambda_{1s}^* = 0$ will not be an equilibrium given the assumptions made.

Finally, I show that the case $\lambda_{1h}^* = 0, \lambda_{1l}^* > 0$ ($z_{1h}^* = z_0^* > z_{1l}^*$) is impossible. $\lambda_{1l}^* = z_0^* - z_{1l}^*$ and $z_{1h}^* = \frac{A + (1 - \theta)(1 - \gamma)}{1 - \theta(1 - \gamma)}$ and

$$z_{1l}^* = \frac{A + (1 - \theta)(1 - \gamma) - z_{1h}^* \theta \frac{1}{N} F''(k_{1l}^T) k_0}{\varpi(k_{1l}^T)} \quad (\text{A.1.23})$$

where $\varpi(k_{1l}^T) = \left[\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - \theta(1 - \gamma) - \theta \frac{1}{N} F''(k_{1l}^T) k_0 \right] > 0$.

In order for $z_{1h}^* > z_{1l}^*$, it will have to be the case $z_{1h}^* > \frac{A + (1 - \theta)(1 - \gamma)}{\varpi(k_{1l}^T)}$ which implies

$$\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - 1 > 0 \quad (\text{A.1.24})$$

Re-writing the first order condition with respect to k_0 and using the fact that $z_{1h}^* = z_0^*$

$$z_0^* = \frac{\pi_l z_{1l}^* (\varpi(k_{1l}^T) + \theta(1 - F'(k_{1l}^T)) + a_{1l} - \gamma)}{[1 - \pi_l \theta(F'(k_{1l}^T) - \gamma + \frac{1}{N} F''(k_{1l}^T) k_0) - \pi_h(1 + a_{1h} - \gamma)]} \quad (\text{A.1.25})$$

where z_{1l}^* is given by

$$z_{1l}^* = \frac{A + (1 - \theta)(1 - \gamma) - z_0^* \theta \frac{1}{N} F''(k_{1s}^T) k_0}{\varpi(k_{1l}^T)} \quad (\text{A.1.26})$$

Combining equations A.1.25 and A.1.26 one can solve for z_0^*

$$z_0^* = \frac{\pi_l [A + (1 - \theta)(1 - \gamma)] \left[1 + \frac{[\theta(1 - F'(k_{1l}^T)) + a_{1l} - \gamma]}{\varpi(k_{1l}^T)} \right]}{\left[1 - \pi_l \theta(F'(k_{1l}^T) - \gamma) - \pi_h(1 + a_{1h} - \gamma) + \pi_l \theta \frac{1}{N} F''(k_{1s}^T) k_0 \frac{[\theta(1 - F'(k_{1l}^T)) + a_{1l} - \gamma]}{\varpi(k_{1l}^T)} \right]} \quad (\text{A.1.27})$$

Next I prove that given Assumption 1.2 and Assumption 1.3 the inequality A.1.24, it will be impossible that $z_0^* = z_{1h}^* = \frac{A + (1 - \theta)(1 - \gamma)}{1 - \theta(1 - \gamma)}$ where z_0^* is given by equation A.1.27. Equating $z_0^* = z_{1h}^*$ and simplifying one gets

$$0 < LHS = \left[\sum_s \pi_s (1 + a_{1s} - \gamma) \right] - 1 = \left(\frac{\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - 1}{\varpi(k_{1l}^T)} \right) \pi_l [\theta(1 - F'(k_{1l}^T)) + a_{1l} - \gamma] = RHS < 0$$

where $[\theta(1 - F'(k_{1l}^T)) + a_{1l} - \gamma] < [\theta(1 - \gamma) + a_{1l} - \gamma] < 0$. Since it is impossible for both $LHS > 0$ and $RHS < 0$, the case $\lambda_{1h}^* = 0, \lambda_{1l}^* > 0$ will never be an equilibrium outcome. Next, I consider the remaining two cases which can occur in equilibrium.

Type 1 equilibrium: $\lambda_{1h}^* > 0, \lambda_{1l}^* = 0$ ($z_{1l}^* = z_0^* > z_{1h}^*$) Using the fact that $\lambda_{1h}^* = z_0^* - z_{1h}^*$ and rewriting equations A.1.5 and A.1.6, one can show that $z_{1h}^* = \frac{A + (1 - \theta)(1 - \gamma)}{1 - \theta(1 - \gamma)}$,

$$z_0^* = \frac{\pi_h z_{1h}^* [(1 - \gamma)(1 - \theta) + a_{1h}] + \pi_l (A + (1 - \theta)(1 - \gamma)) + \pi_l (a_{1l} + \theta(1 - \gamma) - \gamma) z_{1l}^*}{[1 - \pi_h \theta(1 - \gamma)]} \quad (\text{A.1.28})$$

and

$$z_{1l}^* = \frac{A + (1 - \theta)(1 - \gamma)}{\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - \theta(1 - \gamma)}. \quad (\text{A.1.29})$$

Using the equations above, one can solve for k_{1l}^T . In order for $z_{1l}^* > z_{1h}^*$, it has to be the case that $z_{1l}^* > \frac{A+(1-\theta)(1-\gamma)}{1-\theta(1-\gamma)}$ which implies

$$\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - 1 < 0 \quad (\text{A.1.30})$$

The rest of the endogenous variables are given by the following system of equations.

$$k_0 = \frac{\pi_l B_l + n + [F'(k_{1l}^T) - \theta(1-\gamma)] k_{1l}^T \pi_l}{(1 - \pi_h \theta(1-\gamma) + (\gamma - a_{1l} - \theta(1-\gamma)) \pi_l)} \quad (\text{A.1.31})$$

$$k_{1l} = k_0 - k_{1l}^T \quad (\text{A.1.32})$$

$$k_{1h} = \frac{((1-\theta)(1-\gamma) + a_{1h}) k_0}{[1 - \theta(1-\gamma)]} \quad (\text{A.1.33})$$

$$d_{2s} = \theta(1-\gamma) k_{1s}; \quad d_{1l} = \frac{1}{\pi_l} [(k_0 - n) - \pi_h \theta(1-\gamma) k_0] \quad (\text{A.1.34})$$

$$d_{1h} = \theta(1-\gamma) k_0 \quad (\text{A.1.35})$$

and $B_l = (\delta')^{-1} (z_{1l}^{1,P} (k_{1l}^T) - 1)$. Finally $p_{1s} = p_{2s} = 1$ and $q_{1h} = 1, q_{1l} = F'(k_{1l}^T)$.

Type 2 equilibrium: $\lambda_{1s}^* > 0$ ($z_0^* > z_{1s}^*$)

It will be the case that $\lambda_{1s}^* = z_0^* - z_{1s}^*$ and using the first order conditions with respect to k_0 and k_{1s} , equations A.1.5 and A.1.6, one can solve for z_{1s}^* and z_0^* as a function of k_{1l}^T . The rest of the endogenous variables $d_{2s}, d_{1s}, k_{1l}, k_{1h}$ and B_l are pinned down by the falling system of equations: $k_{1l}^{T,\max}$ is determined by the solution to the equation $H(k_{1l}^{T,\max}; k_0 = k_0^{\max}) = 0$ where $H(\cdot)$ is given by equation A.1.19. k_0^{\max} is given by equation A.1.18. Also $d_{1s} = \theta(q_{1s} - \gamma) k_0, d_{2s} = \theta(1-\gamma) k_{1s}$ and

$$\begin{aligned} k_{1l} &= \frac{((F'(k_{1l}^T) - \gamma)(1-\theta) + a_{1l}) k_0 + B_l}{[F'(k_{1l}^T) - \theta(1-\gamma)]} \\ k_{1h} &= \frac{((1-\gamma)(1-\theta) + a_{1h}) k_0}{[1 - \theta(1-\gamma)]} \end{aligned}$$

and $B_l = (\delta')^{-1} (z_{1l}^{1,P} (k_{1l}^T) - 1)$. Finally $p_{1s} = p_{2s} = 1$ and $q_{1h} = 1, q_{1l} = F'(k_{1l}^T)$.

PART 2) Existence and Uniqueness

The proof of existence and uniqueness proceeds in two steps.

Step 1) One can solve for the equilibrium by solving for k_0 . First, I show that for every $k_0 \in [0, k_0^{\max}]$ there exists a unique k_{1l}^T . I will consider two regions for k_0 separately. If the equilibrium k_0 is such that $k_0 \in [0, \hat{k}_0]$ then there will be no fire sale, $k_{1l}^T = 0$, where I will derive \hat{k}_0 as a function of exogenous variables. If the equilibrium k_0 is such that $k_0 \in (\hat{k}_0, k_0^{\max}]$ then there will be a fire sale $k_{1l}^T > 0$ and k_{1l}^T is unique. Also I prove that $\frac{\partial k_{1l}^T(k_0)}{\partial k_0} > 0$.

Step 2) Prove existence and uniqueness using Step 1.

Step 1) Since I proved that the only possible case is $\lambda_{1h} > 0, \lambda_{2s} > 0$ (which encompasses the Type 1 and 2 equilibria), from the period zero budget constraint and the period zero borrowing constraint

$$d_{1l}(k_{1l}^T; k_0) = \min \left\{ \frac{1}{\pi_l} [k_0 [1 - \pi_h \theta (1 - \gamma)] - n], \theta (F'(k_{1l}^T) - \gamma) k_0 \right\}$$

From the budget constraint in the low state in $t = 1$, define

$$H(k_{1l}^T; k_0) = (a_{1l} - \gamma + \theta(1 - \gamma)) k_0 + (F'(k_{1l}^T) - \theta(1 - \gamma)) k_{1l}^T + B_l(k_{1l}^T) - d_{1l}(k_{1l}^T; k_0)$$

where $B_l(k_{1l}^T)$ implies that the bail-out is a function of the fire sale in the low state. Next, consider how the function $H(k_{1l}^T; k_0)$ behaves in the range $k_{1l}^T \in [0, k_0]$. First I show that $H(k_{1l}^T = k_0; k_0) > 0$ for every k_0

$$\begin{aligned} H(k_{1l}^T = k_0; k_0) &= (a_{1l} - \gamma + F'(k_0)) k_0 + B_l(k_0) - d_{1l}(k_0; k_0) \\ &\geq (a_{1l} + (F'(k_0) - \gamma)(1 - \theta)) k_0 + B_l(k_0) > 0 \end{aligned}$$

where for the first inequality I used the fact that $d_{1l}(k_{1l}^T; k_0) \leq \theta(q_{1l} - \gamma) k_0$. Next I show that $H(k_{1l}^T = 0; k_0) > 0$ if $k_0 \in [0, \hat{k}_0]$ and $H(k_{1l}^T = 0; k_0) < 0$ if $k_0 \in (\hat{k}_0, k_0^{\max}]$.

Since I already showed that if $k_0 = k_0^{\max}$, $H(k_{1l}^T = 0; k_0 = k_0^{\max}) < 0$ (inequality A.1.20), here consider only the case $k_0 \in [0, k_0^{\max})$, which implies that $d_{1l}(k_{1l}^T; k_0) = \frac{1}{\pi_l} [k_0 [1 - \pi_h \theta (1 - \gamma)] - n]$

$$\begin{aligned} H(k_{1l}^T = 0; k_0) &= (a_{1l} - \gamma + \theta(1 - \gamma)) k_0 + B_l(0) - \frac{1}{\pi_l} [k_0 [1 - \pi_h \theta (1 - \gamma)] - n] \\ &= \frac{1}{\pi_l} [\{(a_{1l} - \gamma) \pi_l - [1 - \theta(1 - \gamma)]\} k_0 + n] + (\delta')^{-1} \left(\frac{A - \gamma}{1 - \theta(1 - \gamma)} \right) \end{aligned}$$

Since $(a_{1l} - \gamma) \pi_l - [1 - \theta(1 - \gamma)] < 0$, $\frac{\partial H(k_{1l}^T=0; k_0)}{\partial k_0} < 0$. One can show that

$$H(k_{1l}^T = 0; k_0) \begin{cases} > 0 \text{ if } k_0 \leq \hat{k}_0 \\ < 0 \text{ if } k_0 > \hat{k}_0 \end{cases}$$

where

$$\hat{k}_0 = \frac{n + \pi_l (\delta')^{-1} \left(\frac{A - \gamma}{1 - \theta(1 - \gamma)} \right)}{[1 - \theta(1 - \gamma)] + (\gamma - a_{1l}) \pi_l} \quad (\text{A.1.36})$$

I prove that $H(k_{1l}^T; k_0)$ is continuous in k_{1l}^T and $\frac{\partial H(k_{1l}^T; k_0)}{\partial k_{1l}^T} > 0$. The continuity of $H(k_{1l}^T; k_0)$ follows from $B_l(k_{1l}^T)$ and $F'(k_{1l}^T)$ being continuous with respect to k_{1l}^T . From Assumption 1.4 and Lemma A.1.1,

$$\frac{\partial H(k_{1l}^T; k_0)}{\partial k_{1l}^T} = \begin{cases} F'(k_{1l}^T) - \theta(1 - \gamma) + F''(k_{1l}^T) k_{1l}^T + \frac{\partial B_l(k_{1l}^T)}{\partial k_{1l}^T} > 0 \text{ if } k_{1l}^T > 0 \\ 0 \text{ if } k_{1l}^T = 0 \end{cases}$$

Since In the region $k_0 \in [0, \hat{k}_0]$, $H(k_{1l}^T = 0; k_0) > 0$ and $H(k_{1l}^T = k_0; k_0) > 0$ and since $\frac{\partial H(k_{1l}^T; k_0)}{\partial k_{1l}^T} \geq 0$, it follows that $k_{1l}^T(k_0) = 0$ if $k_0 \in [0, \hat{k}_0]$. In the region $k_0 \in (\hat{k}_0, k_0^{\max}]$, $H(k_{1l}^T = 0; k_0) < 0$, $H(k_{1l}^T = k_0; k_0) > 0$, $\frac{\partial H(k_{1l}^T; k_0)}{\partial k_{1l}^T} > 0$ and $H(k_{1l}^T; k_0)$ is continuous. As a result, there exists a unique $k_{1l}^T(k_0) > 0$ if $k_0 \in (\hat{k}_0, k_0^{\max}]$. This completes the proof that for every $k_0 \in [0, k_0^{\max}]$ there exists a unique $k_{1l}^T \geq 0$.

I totally differentiate $H(k_0) = 0$ with respect to k_0 to solve for $\frac{\partial k_{1l}^T(k_0)}{\partial k_0}$ in the relevant range $k_0 \in (\hat{k}_0, k_0^{\max}]$ where there is a fire sale. (This is the only relevant range since I already proved that given the assumptions made, there is a fire sale in the low state in $t = 1$.) In that range, for a given k_0 , k_{1l}^T is pinned down by setting $H(k_{1l}^T; k_0) = 0$.

Totally differentiate $H(k_{1l}^T; k_0) = 0$ with respect to k_0 . From Lemma A.1.1 and also from Assumption 1.3 and Assumption 1.4

$$\frac{\partial k_{1l}^T(k_0)}{\partial k_0} = \begin{cases} \frac{\frac{1}{\pi_l} [1 - \pi_h \theta(1 - \gamma)] - (a_{1l} - \gamma + \theta(1 - \gamma))}{F'(k_{1l}^T) - \theta(1 - \gamma) + F''(k_{1l}^T) k_{1l}^T + B_l'(k_{1l}^T)} > 0 \text{ if } k_0 \in (\hat{k}_0, k_0^{\max}] \\ 0 \text{ if } k_0 \in [0, \hat{k}_0] \end{cases}$$

Step 2) I already proved that given the assumptions made, the only two types of equilibria are an equilibrium of Type 1 (interior equilibrium) and Type 2 (corner equilibrium). In order to prove existence and uniqueness I define the following function

$$\psi^*(k_0) = z_{1l}^*(k_0^T) - z_0^*(k_0)$$

where $z_{1l}^*(k_0^T)$ and $z_0^*(k_0)$ are the marginal value of wealth in the crisis state and in period zero as perceived by the banker and as defined in the equilibrium of Type 1 (equations A.1.29 and A.1.28).

I will prove that $\psi^*(k_0)$ is strictly increasing and crosses the zero line at most once. Let's start with the proof that $\psi^*(k_0)$ is strictly increasing and crosses the zero line at most once. In Step 1, I proved that there is a one-to-one mapping from k_{1l}^T to k_0 and one can solve for k_0 as a function of k_{1l}^T using equation A.1.31 if the equilibrium is of Type 1. Also if there is a fire sale in the crisis state, which is the relevant region, $\frac{\partial k_{1l}^T(k_0)}{\partial k_0} > 0$. Since both z_{1l}^* and z_0^* are functions only of k_{1l}^T which, in turn, is a function of k_0 , I can re-write $\psi^*(k_{1l}^T(k_0))$ as

$$\psi^*(k_{1l}^T(k_0)) = z_{1l}^*(k_{1l}^T(k_0)) - z_0^*(k_{1l}^T(k_0))$$

Define

$$M(k_{1l}^T) = \frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - 1$$

Inequality A.1.30 implies that if the equilibrium is of Type 1, then $M(k_{1l}^T) < 0$. In order to derive the support of the $\psi^*(k_{1l}^T(k_0))$ function, let's investigate the properties of $M(k_{1l}^T)$ in the range $k_{1l}^T \in [0, k_{1l}^{T,\max}]$ so that we can derive for what values of k_{1l}^T , $M(k_{1l}^T) < 0$.² First I show that given Assumption 1.7 $M'(k_{1l}^T) < 0$.³

² $k_{1l}^{T,\max}$ is pinned down by setting equation A.1.19 equal to zero (I already proved that $k_{1l}^{T,\max}$ exists and is unique)

³

$$\begin{aligned} M'(k_{1l}^T) &= \left(\frac{1}{N} + 1 \right) F''(k_{1l}^T) + \frac{1}{N} F'''(k_{1l}^T) k_{1l}^T + \left[\frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} - \frac{\delta'''(B_l)}{(\delta''(B_l))^2 N} \frac{\partial B_l}{\partial k_{1l}^T} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} \right] \\ &= \left(\frac{1}{N} + 1 \right) F''(k_{1l}^T) - \frac{F''(k_{1l}^T) (1 - 2z_{1l}^{1,P}(k_{1l}^T))}{(F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma))^2} \frac{F''(k_{1l}^T)}{\delta''(B_l) N} \left[4 + \frac{\delta'''(B_l)}{(\delta''(B_l))^2} (1 - 2z_{1l}^{1,P}) \right] < 0 \end{aligned}$$

where

$$\begin{aligned} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} &= \frac{F''(k_{1l}^T) (1 - 2z_{1l}^{1,P}(k_{1l}^T))}{F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma)} > 0 \\ \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T \partial k_{1l}^T} &= \frac{-4F''(k_{1l}^T)}{F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma)} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} \\ \frac{\partial B_l(k_{1l}^T)}{\partial k_{1l}^T} &= \frac{1}{\delta''(B_l)} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} \end{aligned}$$

$$\begin{aligned}
M'(k_{1l}^T) &= \left(\frac{1}{N} + 1\right) F''(k_{1l}^T) + \frac{1}{N} F'''(k_{1l}^T) k_{1l}^T + \left[\frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T \partial k_{1l}^T} - \frac{\delta'''(B_l)}{(\delta''(B_l))^2} \frac{\partial B_l}{N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} \right] \\
&= \left(\frac{1}{N} + 1\right) F''(k_{1l}^T) - \frac{F''(k_{1l}^T) \left(1 - 2z_{1l}^{1,P}(k_{1l}^T)\right) \frac{F''(k_{1l}^T)}{\delta''(B_l) N} \left[4 + \frac{\delta'''(B_l)}{(\delta''(B_l))^2} \left(1 - 2z_{1l}^{1,P}\right)\right]}{(F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma))^2} < 0
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} &= \frac{F''(k_{1l}^T) \left(1 - 2z_{1l}^{1,P}(k_{1l}^T)\right)}{F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma)} > 0 \\
\frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T \partial k_{1l}^T} &= \frac{-4F''(k_{1l}^T)}{F''(k_{1l}^T) k_{1l}^T + F'(k_{1l}^T) - \theta(1 - \gamma)} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} > 0 \\
\frac{\partial B_l(k_{1l}^T)}{\partial k_{1l}^T} &= \frac{1}{\delta''(B_l)} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} > 0
\end{aligned}$$

If $M(\cdot) < 0$ for every $k_{1l}^T \in [0, k_{1l}^{T,\max}]$ then $\tilde{k}_{1l}^T = 0$. If $M(\cdot) > 0$ for every $k_{1l}^T \in [0, k_{1l}^{T,\max}]$ then $\tilde{k}_{1l}^T = k_{1l}^{T,\max}$. Otherwise \tilde{k}_{1l}^T is pinned down by $M(\tilde{k}_{1l}^T) = 0$. \tilde{k}_{1l}^T is unique since $M'(k_{1l}^T)$ is strictly decreasing due to Assumption 1.7 and the assumption that $F'''(k_{1l}^T) = 0$.⁴

Therefore the relevant range we need to consider for the $\psi^*(k_{1l}^T(k_0))$ function is $k_{1l}^T \in [\tilde{k}_0, k_0^{\max}]$ where $\tilde{k}_0 = k_0(\tilde{k}_{1l}^T)$ if $0 < \tilde{k}_{1l}^T < k_{1l}^{T,\max}$, $\tilde{k}_0 = 0$ if $\tilde{k}_{1l}^T = 0$ and $\tilde{k}_0 = k_0^{\max}$ if $\tilde{k}_{1l}^T = k_{1l}^{T,\max}$. Also k_0^{\max} is given by equation A.1.18. Notice that in the case of a continuum of banks, $N \rightarrow \infty$, $\tilde{k}_0 = 0$.

$$\psi^*(k_{1l}^T(k_0)) = z_{1l}^*(k_{1l}^T(k_0)) - z_0^*(k_{1l}^T(k_0)) \text{ if } k_{1l}^T \in [\tilde{k}_0, k_0^{\max}]$$

$$\psi^*(k_{1l}^T(k_0)) = \frac{z_{1l}^*[1 - \theta(1 - \gamma) - \pi_l(a_{1l} - \gamma)] - z_h^*[\pi_h[(1 - \gamma)(1 - \theta) + a_{1h}] + \pi_l(1 - \theta(1 - \gamma))]}{[1 - \pi_h\theta(1 - \gamma)]} \quad (\text{A.1.37})$$

Given Assumption 1.7 which implies $\frac{\partial \left(\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) \right)}{\partial k_{1l}^T} < 0$, I can differentiate equation A.1.29

⁴The assumption $F'''(k_{1l}^T) = 0$ can be relaxed. It would be sufficient that $F'''(k_{1l}^T)$ is fairly small.

$$\frac{\partial z_{1l}^*(k_0)}{\partial k_0} = \frac{-\frac{\partial \left(\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) \right)}{\partial k_{1l}^T} \frac{\partial k_{1l}^T(k_0)}{\partial k_0} z_{1l}^*(k_0)}{\left[\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - \theta(1-\gamma) \right]} > 0$$

Combining equations A.1.29 and A.1.28

$$\begin{aligned} \frac{\partial \psi^*(k_{1l}^T(k_0))}{\partial k_0} &= \frac{\partial z_{1l}^*(k_0)}{\partial k_0} - \frac{\partial z_0^*(k_0)}{\partial k_0} \\ &= \frac{\partial z_{1l}^*(k_0)}{\partial k_0} \left[\frac{1 - \theta(1-\gamma) + \pi_l(\gamma - a_{1l})}{[1 - \pi_h \theta(1-\gamma)]} \right] > 0 \end{aligned}$$

Next, evaluate $\psi^*(k_{1l}^T)$ at $k_{1l}^T = \tilde{k}_{1l}^T$ and $k_{1l}^T = k_{1l}^{T,\max}$ which is the relevant range for the Type 1 equilibrium.

If $k_{1l}^T = \tilde{k}_{1l}^T$, then $z_{1l}^*(\tilde{k}_{1l}^T) = \frac{A + (1-\theta)(1-\gamma)}{1-\theta(1-\gamma)}$. Given Assumption 1.2

$$\psi^*(\tilde{k}_{1l}^T) = \frac{A + (1-\theta)(1-\gamma)}{1-\theta(1-\gamma)} \frac{1 - \sum_s \pi_s (a_{1s} - \gamma + 1)}{1 - \pi_h \theta(1-\gamma)} < 0$$

If $\psi^*(k_{1l}^T) < 0$ for every $k_{1l}^T \in [\tilde{k}_{1l}^T, k_{1l}^{T,\max}]$, then the equilibrium is a corner equilibrium of Type 2. If $\psi^*(k_{1l}^{T,\max}) > 0$, the equilibrium is an interior equilibrium of Type 1. This completes the proof of existence and uniqueness.

Proposition 1.3.2

Proposition 1.3.2: (i) Given Assumptions 1.1-1.7 and the assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, there is never a fire sale in the high state, $q_{1h} = 1$ and there is a fire sale in the low state, $q_{1l} < 1$. (ii) The equilibrium of the constrained Central Planner's problem exists and is unique and is one of the following types: Type 1) $z_0^{CP} = z_{1l}^{CP} > z_{1h}^{CP}$ (interior equilibrium); Type 2) $z_0^{CP} > z_{1s}^{CP}$ (corner equilibrium where the banker borrows to the maximum in $t = 0$) where $z_{1s}^{CP} = z_{1s}^{0,CP}$. The optimal bail-out is determined by

$$1 + \delta'(B_l) = z_{1l}^{1,CP} \quad (\text{A.1.38})$$

(iii) If also the following assumption

$$\begin{aligned} &[\pi_h(a_{1h} - \gamma) + 1 - \theta(1-\gamma)] \frac{A + (1-\gamma)(1-\theta)}{1 - \theta(1-\gamma)} \quad (\text{Assumption 1.9}) \\ &< \frac{F''(k_{1l}^T) k_{1l}^T + A + (1-\gamma)(1-\theta)}{F'(k_{1l}^T) + F''(k_{1l}^T) k_{1l}^T - \theta(1-\gamma)} [\pi_l(\gamma - a_{1l}) + 1 - \theta[1-\gamma]] \quad (\text{A.1.39}) \end{aligned}$$

is satisfied where $k_{1l}^T = k_{1l}^{T,\max}$, the only possible equilibrium is the interior equilibrium of Type 1 where

$$\sum \pi_s \left(-F''(k_{1s}^T) k_{1s}^T + z_{1s}^{CP} (F'(k_{1s}^T) + a_{1s} - \gamma + F''(k_{1s}^T) k_{1s}^T) \right) + \pi_h \lambda_{1h}^{CP} \theta [1 - \gamma] = z_0^{CP} \quad (\text{A.1.40})$$

$$z_{1h}^{CP} = \frac{A + (1 - \theta)(1 - \gamma)}{1 - \theta(1 - \gamma)} \quad (\text{A.1.41})$$

$$z_{1l}^{1,CP} = z_{1l}^{CP} = \frac{F''(k_{1l}^T) k_{1l}^T + A + (1 - \theta)(1 - \gamma)}{F'(k_{1l}^T) - \theta(1 - \gamma) + F''(k_{1l}^T) k_{1l}^T} \quad (\text{A.1.42})$$

and the first order conditions with respect to d_{2s} and d_{1s} imply $\lambda_{2s}^{CP} > 0, \lambda_{1l}^{CP} = 0, \lambda_{1h}^{CP} = z_0^{CP} - z_{1h}^{CP} \geq 0$.

Before I prove Proposition 1.3.2 I prove Lemmas A.1.3 and A.1.4.

Lemma A.1.3 *Conditional on Assumptions 1.1-1.6 and conditional on a fire sale in the low state, $\frac{\partial B_l(k_{1l}^T)}{\partial k_{1l}^T} > 0$ and $\frac{\partial z_{1l}^{1,CP}}{\partial k_{1l}^T} > 0$.*

Proof of Lemma A.1.3. The proof is identical to the proof of Lemma A.1.1 since $z_{1l}^{1,CP}(k_{1l}^T) = z_{1l}^{1,P}(k_{1l}^T)$.

Lemma A.1.4 *Given Assumption 1.5, considering a symmetric equilibrium, there is never a fire sale in the high state, $q_{1h} = 1$. Given Assumption 1.2 and also given the additional Assumption 1.6, it is always the case that there is a fire sale in the low state, $q_{1l} < 1$.*

Proof of Lemma A.1.4. The proof that $q_{1h} = 1$ is identical to the one in Lemma A.1.1. The proof that $q_{1l} < 1$ is very similar since one can re-write the first order conditions of the Central Planner's problem, assuming that $q_{1l} = 1$, as

$$\sum \pi_s \left(-\frac{\partial B_s}{\partial k_0} (1 + \delta'(B_s)) + z_{1s}^{CP} \left(1 + a_{1s} - \gamma + \frac{\partial B_s}{\partial k_0} \right) + \lambda_{1s}^{CP} \theta [1 - \gamma] \right) = z_0^{CP}$$

since $z_{1l}^{1,CP}(k_{1l}^T) = \frac{A + (1 - \theta)(1 - \gamma)}{1 - \theta(1 - \gamma)}$ implies that $\frac{\partial z_{1s}^{1,CP}}{\partial k_{1s}^T} = 0$, $\frac{\partial B_s}{\partial k_0} = \frac{1}{\delta''(B_s)} \frac{\partial z_{1s}^{1,CP}}{\partial k_{1s}^T} = 0$ and $\frac{\partial B_s}{\partial k_{1s}} = -\frac{1}{\delta''(B_s)} \frac{\partial z_{1s}^{1,CP}}{\partial k_{1s}^T} = 0$. As a result,

$$\sum \pi_s \left(z_{1s}^{CP} (1 + a_{1s} - \gamma) + \lambda_{1s}^{CP} \theta [1 - \gamma] \right) = z_0^{CP}$$

where $\frac{A + (1 - \theta)(1 - \gamma)}{1 - \theta(1 - \gamma)} = z_{1s}^{CP}$ and $z_0^{CP} - z_{1s}^{CP} = \lambda_{1s}^{CP}$. The equations above coincide with the equations in the proof of Lemma A.1.2, Step 1. The rest of the proof is identical to the proof in Lemma A.1.2.

Proof of Proposition 1.3.2:

Part 1) First I prove that the only two types of equilibria possible are of Type 1 and 2 and the proof is similar to the proof in Proposition 1.3.1

In order to characterize the equilibrium, I consider all four possible combinations of whether λ_{1h} and λ_{1l} are greater than or equal to zero.

If $\lambda_{1h} = 0, \lambda_{1l} = 0$ then $z_{1h}^{CP} = z_{1l}^{CP} = z_0^{CP}$. Plugging equation A.1.8 in equation A.1.11a one gets $z_{1s}^{CP}(k_{1s}^T) = \frac{F''(k_{1s}^T)k_{1s}^T + A + (1-\theta)(1-\gamma)}{F''(k_{1s}^T)k_{1s}^T + F'(k_{1s}^T) - \theta(1-\gamma)}$. However, since there is a fire sale only in the low state and no fire sale in the high state, then one can prove that $z_{1l}^{CP} = \frac{F''(k_{1l}^T)k_{1l}^T + A + (1-\theta)(1-\gamma)}{F''(k_{1l}^T)k_{1l}^T + F'(k_{1l}^T) - \theta(1-\gamma)} > \frac{A + (1-\theta)(1-\gamma)}{1-\theta(1-\gamma)} = z_{1h}^{CP}$. This is true since $(1 - F'(k_{1l}^T))(A + (1 - \theta)(1 - \gamma)) > (A - \gamma)F''(k_{1l}^T)k_{1l}^T$. Hence it will never be the case that $\lambda_{1h} = 0, \lambda_{1l} = 0$.

If $\lambda_{1h} = 0, \lambda_{1l} > 0$, then $z_{1h}^{CP} = z_0^{CP} = \frac{A + (1-\theta)(1-\gamma)}{1-\theta(1-\gamma)}$ and $z_0^{CP} - \lambda_{1l} = z_{1l}^{CP} < z_{1h}^{CP}$.

$$z_{1l}^{CP} = \frac{F''(k_{1l}^T)k_{1l}^T - \frac{\partial B_l}{\partial k_{1l}}z_{1l}^{1,CP} + A + (1-\gamma)(1-\theta) - z_0^{CP}\theta\tilde{F}''(k_{1l}^T)k_0}{F''(k_{1l}^T)k_{1l}^T - \frac{\partial B_l}{\partial k_{1l}} + F'(k_{1s}^T) - \theta(1-\gamma) - \theta\tilde{F}''(k_{1l}^T)k_0} \quad (\text{A.1.43})$$

I prove by contradiction that it is impossible that $z_{1l}^{CP} < z_{1h}^{CP}$. After plugging in $z_{1l}^{1,CP}$ given by equation A.1.8 in equation A.1.43 and taking into account that $z_{1h}^{CP} = z_0^{CP}$, one can re-write the inequality $z_{1l}^{CP} < z_{1h}^{CP}$ as $\frac{F''(k_{1l}^T)k_{1l}^T + A + (1-\gamma)(1-\theta)}{F'(k_{1l}^T) - \theta(1-\gamma) + F''(k_{1l}^T)k_{1l}^T} < z_{1h}^{CP} = \frac{A + (1-\theta)(1-\gamma)}{1-\theta(1-\gamma)}$ which is a contradiction. As a result, it is impossible that $\lambda_{1h} = 0, \lambda_{1l} > 0$.

Type 1 equilibrium: $\lambda_{1h} > 0, \lambda_{1l} = 0$ ($z_0^{CP} = z_{1l}^{CP} > z_{1h}^{CP}$)

Notice that $z_0^{CP} = z_{1l}^{CP} = z_{1h}^{CP} + \lambda_{1h}^{CP} > z_{1h}^{CP}$ and from the first order condition with respect to k_{1h} , $z_{1h}^{CP} = \frac{A + (1-\theta)(1-\gamma)}{1-\theta(1-\gamma)}$

$$z_0^{CP} = \frac{\pi_h z_{1h}^{CP}((1-\gamma)(1-\theta) + a_{1h}) + \pi_l(A + (1-\theta)(1-\gamma) + z_{1l}^{CP}[\theta(1-\gamma) + a_{1l} - \gamma])}{(1 - \pi_h\theta[1 - \gamma])} \quad (\text{A.1.44})$$

Plugging in for $z_{1l}^{1,CP}$, from the first order condition with respect to k_{1l}

$$z_{1l}^{1,CP} = z_{1l}^{CP} = \frac{F''(k_{1l}^T)k_{1l}^T + A + (1-\theta)(1-\gamma)}{F'(k_{1l}^T) - \theta(1-\gamma) + F''(k_{1l}^T)k_{1l}^T} \quad (\text{A.1.45})$$

The rest of the variables are determined by the same set of equations as the ones in the Type 1 equilibrium in Proposition 1.3.1. The equilibrium of Type 1 is possible since we already proved that $z_{1l}^{CP} > z_{1h}^{CP}$.

Type 2 equilibrium: If $\lambda_{1h} > 0, \lambda_{1l} > 0$ ($z_0^{CP} > z_{1l}^{CP} > z_{1h}^{CP}$)

Since I already proved that $\lambda_{2s} > 0$, in this type of equilibrium the banker borrows to the maximum in $t = 0$.

Next I prove existence and uniqueness.

As in the proof of Proposition 1.3.1, one can show that for every $k_0 \in [0, k_0^{\max}]$ there exists a unique k_{1l}^T and if the equilibrium k_0 is such that $k_0 \in (\hat{k}_0, k_0^{\max}]$ then there will be a fire sale, $k_{1l}^T > 0$, where \hat{k}_0 is pinned down by equation A.1.36. Also as in Proposition 1.3.1, one can prove that $\frac{\partial k_{1l}^T(k_0)}{\partial k_0} > 0$ if $k_0 \in (\hat{k}_0, k_0^{\max}]$.

I take into account that it will be always the case that $\lambda_{th}^{CP} > 0, \lambda_{2l}^{CP} > 0$. Consider the interior equilibrium of Type 1 which implies $\lambda_{1l}^{CP} = 0$. Following the same steps as in the Proof of Proposition 1.3.1, define the following function which will be used to pin down the equilibrium k_0

$$\psi^{CP}(k_0) = z_{1l}^{CP}(k_0) - z_0^{CP}(k_0) \text{ if } k_0 \in [\hat{k}_0, k_0^{\max}] \quad (\text{A.1.46})$$

where $z_0^{CP}(k_0)$ and $z_{1l}^{CP}(k_0)$ are given by equations A.1.44 and A.1.45. If $\psi^{CP}(k_0) = 0$, then the equilibrium is interior and of Type 1. If for every k_0 in the range $[\hat{k}_0, k_0^{\max}]$, $\psi^{CP}(k_0) < 0$ then the equilibrium is a corner equilibrium where it is optimal to borrow to the maximum in period zero against the high and the low states. I will show that given the assumptions made, it will be never the case that $\psi^{CP}(k_0) > 0$ for all $k_0 \in [\hat{k}_0, k_0^{\max}]$. Since $\frac{\partial k_{1l}^T(k_0)}{\partial k_0} > 0$ and since $z_{1l}^{CP} = z_{1l}^{1,CP}$ in the interior equilibrium.

$$\frac{\partial z_{1l}^{CP}(k_0)}{\partial k_0} = \frac{\partial k_{1l}^T(k_0)}{\partial k_0} \frac{z_{1l}^{CP}}{\partial k_{1l}^T} > 0$$

where $\frac{z_{1l}^{CP}}{\partial k_{1l}^T} = \frac{\partial z_{1l}^{1,CP}}{\partial k_{1l}^T} > 0$ and is given by equation A.1.15

$$\psi^{CP}(k_0) = \frac{z_{1l}^{CP}(k_0)(1 - \theta[1 - \gamma] - \pi_l[a_{1l} - \gamma]) - \pi_h z_{1h}^{CP}((1 - \gamma)(1 - \theta) + a_{1h}) - \pi_l(A + (1 - \theta)(1 - \gamma))}{(1 - \pi_h \theta[1 - \gamma])}$$

$$\frac{\partial \psi^{CP}(k_0)}{\partial k_0} = \frac{\partial z_{1l}^{CP}(k_0)}{\partial k_0} \left[\frac{1 - \theta(1 - \gamma) + \pi_l(\gamma - a_{1l})}{[1 - \pi_h \theta(1 - \gamma)]} \right] > 0$$

Next I show that given the assumptions made, it will be never the case that $\psi^*(k_0) > 0$ for all $k_0 \in [\hat{k}_0, k_0^{\max}]$. I already proved that $\frac{\partial \psi^*(k_0)}{\partial k_0} > 0$. As a result, it's sufficient to prove that $\psi^*(k_0 = \hat{k}_0) < 0$. Since by definition if $k_0 = \hat{k}_0$, there will be no fire sale in the low state,

$\psi^{CP}(k_0 = \hat{k}_0) = \frac{z_{1h}^{CP}[1 - \sum_s((1 - \gamma)(1 - \theta) + a_{1s}) - \theta[1 - \gamma]]}{(1 - \pi_h \theta[1 - \gamma])} < 0$. As a result, the only possible equilibria are either borrow to the maximum against both states in period zero ($z_0^* > z_{1l}^* > z_{1h}^*$) or the interior equilibrium ($z_0^* = z_{1l}^* > z_{1h}^*$). Given that $\psi'(k_0) > 0$ in the relevant range $k_0 \in [\hat{k}_0, k_0^{\max}]$, the interior equilibrium exists and is unique.

Finally, I prove that Assumption 1.9 ensures that an equilibrium of Type 2 never occurs. Consider the case where $\lambda_{ts} > 0$ and assume that Assumption 1.6 is satisfied which implies that even if the equilibrium is of Type 2 there will be a fire sale in the crisis state. It is sufficient to show that given Assumption 1.9, it is always the case that $z_{1l}^{0,CP} = z_{1l}^{CP} > z_0^{CP}$ and, as a result, equilibrium of Type 2 will never occur. One can show that $z_{1l}^{CP} - z_0^{CP} > 0$ by re-writing the first order conditions of the Central Planner

$$z_0^{CP} = \frac{\pi_l \left(-F''(k_{1l}^T) k_{1l}^T - \frac{\partial B_l}{\partial k_0} z_{1l}^{1,CP} + z_{1l}^{0,CP} \left(\begin{array}{c} F'(k_{1l}^T) + a_{1l} - \gamma + F''(k_{1l}^T) k_{1l}^T \\ + \frac{\partial B_l}{\partial k_0} - \theta [F'(k_{1l}^T) - \gamma + F''(k_{1l}^T) k_0] \end{array} \right) \right)}{(1 - \pi_h \theta [1 - \gamma] - \pi_l \theta [F'(k_{1l}^T) - \gamma + F''(k_{1l}^T) k_0])} \quad (\text{A.1.47})$$

$$F''(k_{1l}^T) k_{1l}^T + A + (1 - \gamma)(1 - \theta) \quad (\text{A.1.48a})$$

$$+ z_{1l}^{0,CP} \left[-F''(k_{1l}^T) k_{1l}^T + \frac{\partial B_l}{\partial k_{1l}} - (F'(k_{1l}^T) - \theta(1 - \gamma)) \right] - (z_0^{CP} - z_{1l}^{0,CP}) \theta \tilde{F}''(k_{1l}^T) k_0 = \frac{\partial B_l}{\partial k_{1l}} z_{1l}^{1,CP}$$

$$(z_0^{CP} - z_{1l}^{0,CP}) = \frac{[\pi_h(a_{1h} - \gamma) + 1 - \theta(1 - \gamma)] \frac{A + (1 - \gamma)(1 - \theta)}{1 - \theta(1 - \gamma)} + z_{1l}^{0,CP} [\pi_l(a_{1l} - \gamma) - 1 + \theta[1 - \gamma]]}{(1 - \pi_h \theta [1 - \gamma] - \pi_l \theta [F'(k_{1l}^T) - \gamma])} < 0 \quad (\text{A.1.49})$$

if

$$[\pi_h(a_{1h} - \gamma) + 1 - \theta(1 - \gamma)] \frac{A + (1 - \gamma)(1 - \theta)}{1 - \theta(1 - \gamma)} < z_{1l}^{0,CP} [\pi_l(\gamma - a_{1l}) + 1 - \theta[1 - \gamma]]$$

Since $z_{1l}^{0,CP} > z_{1l}^{1,CP} = \frac{F''(k_{1l}^T) k_{1l}^T + A + (1 - \gamma)(1 - \theta)}{F'(k_{1l}^T) + F''(k_{1l}^T) k_{1l}^T - \theta(1 - \gamma)}$, a sufficient condition is given by Assumption 1.9.

Corollary 1.4.1

Corollary 1.4.1: *Conditional on Assumptions 1.1-1.7 and the assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, also conditional on an interior equilibrium for the Central Planner (Assumption 1.9 is satisfied) and given Assumption 1.10, (a sufficient and necessary condition only for the $N < \infty$ case)*

$$\frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} z_{1l}^{CP} > \left(\left(1 - \frac{1}{N} \right) z_{1l}^{CP} - 1 \right) F''(k_{1l}^T) k_{1l}^T \quad \text{Assumption 10}$$

the Central Planner values an extra dollar in the hands of the banker in the crisis state by more than the banker in the decentralized equilibrium does, for a given k_0 .

$$z_{1l}^{CP} > z_{1l}^*$$

Proof of Corollary 1.4.1:

$$\begin{aligned} z_{1l}^{CP} &= \frac{F''(k_{1l}^T) k_{1l}^T + A + (1 - \theta)(1 - \gamma)}{F'(k_{1l}^T) - \theta(1 - \gamma) + F''(k_{1l}^T) k_{1l}^T} \\ &> \frac{A + (1 - \theta)(1 - \gamma)}{\frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - \theta(1 - \gamma)} = z_{1l}^* \end{aligned}$$

which can be re-written as

$$\left[\left(\frac{1}{N} - 1 \right) F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l)N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} \right] z_{1l}^{CP} > -F''(k_{1l}^T) k_{1l}^T$$

If $N \rightarrow \infty$, $z_{1l}^{CP} > z_{1l}^*$ is always true since $z_{1l}^{CP} > 1$. If $N < \infty$, then in order for $z_{1l}^{CP} > z_{1l}^*$, Assumption 1.10 has to be satisfied.

If $N = 2$ and the country has zero fiscal capacity a sufficient condition is $2 < A + (1 - \gamma)(1 + \theta)$ since

$$F''(k_{1l}^T) k_{1l}^T + 2F'(k_{1l}^T) < 2 < A + (1 - \gamma)(1 + \theta)$$

Since there is a one to one mapping between k_0 and k_{1l}^T , the result is true for a given k_0 as well.

Proposition 1.5.1

Proposition 1.5.1: *Given Assumptions 1.1-1.8, Assumption 1.10 and the assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, for a given exogenous minimum bank capital requirement such that $\rho > \frac{n}{k_0^s(\rho=0)}$ and considering a symmetric equilibrium, the decentralized equilibrium can be one of the following four types:*

Type 1) $z_{1l}^*(k_{1l}^T(\rho)) = z_0^*(k_{1l}^T(\rho)) > z_{1h}^*$ if $k_{1l}^T(\rho) \in [\tilde{k}_{1l}^T, \hat{k}_{1l}^{T,\max})$

Type 2) $z_0^*(k_{1l}^T(\rho)) > z_{1s}^*(k_{1l}^T(\rho))$ if $k_{1l}^T(\rho) = k_{1l}^{T,\max}$

Type 3) $z_{1l}^*(k_{1l}^T(\rho)) = z_0^*(k_{1l}^T(\rho)) = z_{1h}^*$ if $k_{1l}^T(\rho) = \tilde{k}_{1l}^T$

Type 4) $z_{1h}^* = z_0^*(k_{1l}^T(\rho)) > z_{1l}^*(k_{1l}^T(\rho))$ if $k_{1l}^T(\rho) \in [0, \tilde{k}_{1l}^T)$ where $k_{1l}^{T,\max}$ is determined in Section A.1.4

in the Appendix. \tilde{k}_{1l}^T is unique and exists and if $0 < \tilde{k}_{1l}^T < k_{1l}^{T,\max}$, \tilde{k}_{1l}^T is determined by $M(\tilde{k}_{1l}^T) = 0$ where

$$M(k_{1l}^T) = \frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - 1.$$

$k_0^*(\rho = 0)$ is the optimal period zero investment chosen by the banker if there is no minimum capital requirement.

Proof of Proposition 1.5.1.: The proof that \tilde{k}_{1l}^T is unique and exists is provided in the proof of Proposition 1.3.1. Since the minimum capital requirement constraint is binding, k_0 is pinned down by $k_0(\rho) = \frac{n}{\rho}$. Let's consider the different types of equilibria.

Equilibrium of Type 1: If $\lambda_{1h}^* > 0, \lambda_{1l}^* = 0$ ($z_{1l}^* = z_0^* > z_{1h}^*$)

$k_{1l}^T(\rho)$ is pinned down from the budget constraint in the low state in $t = 1$

$$LHS = [1 - \theta(1 - \gamma) + \pi_l(\gamma - a_{1l})] \frac{n}{\rho} - n = \quad (A.1.50)$$

$$\pi_l [k_{1l}^T (F'(k_{1l}^T) - \theta(1 - \gamma)) + B_l(k_{1l}^T)] = RHS(k_{1l}^T) \quad (A.1.51)$$

$$\frac{\partial RHS}{\partial k_{1l}^T} = \pi_l [F'(k_{1l}^T) - \theta(1 - \gamma) + k_{1l}^T F''(k_{1l}^T) + B_l'(k_{1l}^T)] > 0$$

$$RHS(0) = B_l(0) = (\delta')^{-1} \left(\frac{A - \gamma}{1 - \theta(1 - \gamma)} \right) > 0$$

$$\lim_{k_{1l}^T \rightarrow \infty} RHS(k_{1l}^T) \rightarrow \infty$$

Notice that if ρ is such that there is a fire-sale in the crisis state which are the only equilibria considered, $LHS > RHS(0)$ and the equilibrium will exist. The rest of the equations are:

$$k_{1l}(\rho) = k_0(\rho) - k_{1l}^T(\rho)$$

$$d_{1h}(\rho) = \theta(1 - \gamma) k_0(\rho); \quad d_{1l}(\rho) = \frac{1}{\pi_l} [k_0(\rho) - n - \pi_h \theta(1 - \gamma) k_0(\rho)]$$

$$d_{2s}(\rho) = \theta(1 - \gamma) k_{1s}(\rho)$$

$$k_{1h}(\rho) = \frac{((1-\theta)(1-\gamma) + a_{1h})k_0(\rho)}{[1-\theta(1-\gamma)]}$$

k_{1l}^T in the Type 1 equilibrium is determined by equation A.1.50 where the condition $z_{1l}^* > z_{1h}^*$ has to be satisfied which implies $M(k_{1l}^T) = \frac{1}{N}F''(k_{1l}^T)k_{1l}^T + \frac{1}{\delta''(B_l)N}\frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - 1 < 0$ (see the proof of Proposition 1.3.1) and also the borrowing constraint in the low state needs to be not binding i.e.

$$d_{1l}(\rho) = \frac{1}{\pi_l} \left[(1 - \pi_h \theta (1 - \gamma)) \frac{n}{\rho} - n \right] < \theta (F'(k_{1l}^T) - \gamma) \frac{n}{\rho}$$

$$1 - \pi_h \theta (1 - \gamma) - \theta \pi_l (F'(k_{1l}^T) - \gamma) < \rho$$

Equilibrium of Type 2: If $\lambda_{1s}^* > 0$ ($z_0^* > z_{1s}^*$)

This will be the optimal equilibrium only if $\rho = \frac{n}{k_0^{\max}}$. The rest of the equations are the same as the equations in the equilibrium of Type 2 in Proposition 1.3.1.

Equilibrium of Type 3: If $\lambda_{1s}^* = 0$ ($z_{1l}^* = z_0^* = z_{1h}^*$)

\tilde{k}_{1l}^T is pinned down by $M(\tilde{k}_{1l}^T) = 0$ and $k_{1l}(\rho) = \frac{n}{\rho} - \tilde{k}_{1l}^T$

$$d_{1l}(\rho) = \left(F'(\tilde{k}_{1l}^T) + a_{1l} - \gamma \right) \frac{n}{\rho} + B_l + \left[\theta(1-\gamma) - F'(\tilde{k}_{1l}^T) \right] k_{1l}(\rho)$$

$$d_{1h}(\rho) = \frac{\frac{n}{\rho} - n - \pi_l d_{1l}}{\pi_h}$$

$$k_{1h}(\rho) = \frac{d_{1h} - (1 + a_{1h} - \gamma) \frac{n}{\rho}}{[\theta(1-\gamma) - 1]}$$

In order for the equilibrium to be of Type 3 the borrowing constraints in $t = 0$ against the high and low state should not be binding. One has to check that

$$d_{1l} < \theta \left(F'(\tilde{k}_{1l}^T) - \gamma \right) k_0$$

$$d_{1h} < \theta(1-\gamma)k_0$$

Equilibrium of Type 4: If $\lambda_{1h}^* = 0, \lambda_{1l}^* > 0$ ($z_{1h}^* = z_0^* > z_{1l}^*$)

k_{1l}^T is pinned down by the budget constraint in $t = 1$ in the low state

$$0 = (a_{1l} - \gamma + \theta (1 - F' (k_{1l}^T))) \frac{n}{\rho} + B_l (k_{1l}^T) - k_{1l}^T [\theta (1 - \gamma) - F' (k_{1l}^T)]$$

The rest of the equations are given by

$$d_{1l} = \theta (F' (k_{1l}^T (\rho)) - \gamma) \frac{n}{\rho}$$

$$d_{1h} = \frac{1}{\pi_h} \left[\frac{n}{\rho} (1 - \pi_l \theta (F' (k_{1l}^T (\rho)) - \gamma)) - n \right]$$

$$k_{1h} = \frac{(1 + a_{1h} - \gamma) \frac{n}{\rho} - d_{1h}}{1 - \theta (1 - \gamma)}$$

In order for the equilibrium to be of Type 4, the following conditions also have to be satisfied, $z_0^* > z_{1l}^*$ which implies $M (k_{1l}^T) > 0$ and also $d_{1h} < \theta (1 - \gamma) k_0$ which implies

$$\frac{n}{\rho} (1 - \pi_l \theta (F' (k_{1l}^T (\rho)) - \gamma) - \pi_h \theta (1 - \gamma)) - n < 0$$

Corollary 1.5.2

Corollary 1.5.2: *If $N < \infty$ and $\chi > 0$, bankers realize that they affect the bail-out received both via k_0^i and d_{1l}^i , i.e. $\frac{\partial B_l^i}{\partial k_0^i} = \frac{\partial B_l^i}{\partial k_{1l}^{i,T}} \frac{\partial k_{1l}^{i,T}}{\partial k_0^i} > 0$ and $\frac{\partial B_l^i}{\partial d_{1l}^i} = \frac{\partial B_l^i}{\partial k_{1l}^{i,T}} \frac{\partial k_{1l}^{i,T}}{\partial d_{1l}^i} > 0$ where $\frac{\partial B_l^i}{\partial k_0^i}$ and $\frac{\partial B_l^i}{\partial d_{1l}^i}$ are total derivatives. Also for a given k_{1l}^T , the fewer the banks are and the larger the fiscal capacity is, the stronger the moral hazard is; $\frac{\partial^2 B_l^i}{\partial k_0^i \partial N} < 0$, $\frac{\partial^2 B_l^i}{\partial k_0^i \partial \chi} > 0$ and $\frac{\partial^2 B_l^i}{\partial d_{1l}^i \partial N} < 0$, $\frac{\partial^2 B_l^i}{\partial d_{1l}^i \partial \chi} > 0$.*

Proof of Corollary 1.5.2. Totally differentiate the budget constraint in the low state with respect to k_0^i holding d_{1l}^i fixed to solve for $\frac{\partial k_{1l}^i}{\partial k_0^i}$

$$\frac{\partial k_{1l}^i}{\partial k_0^i} = \frac{F' (k_{1l}^T) + a_{1l} - \gamma + F'' (k_{1l}^T) k_{1l}^{i,T} \frac{1}{N} + \frac{\partial B_l^i}{\partial k_0^i}}{F' (k_{1l}^T) - \theta (1 - \gamma) + \frac{1}{N} F'' (k_{1l}^T) k_{1l}^{i,T}} \quad (\text{A.1.52})$$

Totally differentiate the budget constraint in the low state with respect to d_{1l}^i holding k_0^i fixed to solve for $\frac{\partial k_{1l}^i}{\partial d_{1l}^i}$

$$\frac{\partial k_{1l}^i}{\partial d_{1l}^i} = \frac{\frac{\partial B_l^i}{\partial d_{1l}^i} - 1}{\left[F' (k_{1l}^T) - \theta (1 - \gamma) + \frac{1}{N} F'' (k_{1l}^T) k_{1l}^{i,T} \right]} \quad (\text{A.1.53})$$

From the first order condition with respect to B_l of the policy maker in the middle period

$$\frac{\partial B_l^i}{\partial k_0^i} = \frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}(k_{1l}^T)}{\partial k_{1l}^T} \frac{\partial k_{1l}^{i,T}}{\partial k_0^i} = \frac{1}{\delta''(B_l^i) N} \frac{\partial z_{1l}^{1,RP}(k_{1l}^T)}{\partial k_{1l}^T} \left(1 - \frac{\partial k_{1l}^i}{\partial k_0^i}\right) \quad (\text{A.1.54})$$

$$\frac{\partial B_l^i}{\partial d_{1l}^i} = \frac{1}{\delta''(B_l) N} \frac{\partial z_{1l}^{1,P}(k_{1l}^T)}{\partial k_{1l}^T} \frac{\partial k_{1l}^{i,T}}{\partial d_{1l}^i} = -\frac{\partial k_{1l}^i}{\partial d_{1l}^i} \frac{1}{\delta''(B_l^i) N} \frac{\partial z_{1l}^{1,RP}(k_{1l}^T)}{\partial k_{1l}^T} \quad (\text{A.1.55})$$

Combining equations A.1.52 and A.1.54, and also equations A.1.53 and A.1.55, and also from Assumption 1.3 and Assumption 1.4.

$$\begin{aligned} \frac{\partial B_l^i}{\partial k_0^i} &= \frac{\frac{\partial z_{1l}^{1,P}(k_{1l}^T)}{\partial k_{1l}^T} [\gamma - \theta(1 - \gamma) - a_{1l}]}{\delta''(B_l) N \left(F'(k_{1l}^T) - \theta(1 - \gamma) + \frac{1}{N} F''(k_{1l}^T) k_{1l}^{i,T} \right) + \frac{\partial z_{1l}^{1,P}(k_{1l}^T)}{\partial k_{1l}^T}} > 0 \\ \frac{\partial B_l^i}{\partial d_{1l}^i} &= \frac{\frac{\partial z_{1l}^{1,P}(k_{1l}^T)}{\partial k_{1l}^T}}{\left(F'(k_{1l}^T) - \theta(1 - \gamma) + \frac{1}{N} F''(k_{1l}^T) k_{1l}^{i,T} \right) \delta''(B_l) N + \frac{\partial z_{1l}^{1,P}(k_{1l}^T)}{\partial k_{1l}^T}} > 0 \end{aligned}$$

Also notice that for a given k_{1l}^T , the fewer the banks are and the larger the fiscal capacity is, the larger the moral hazard is $\frac{\partial^2 B_l^i}{\partial k_0^i \partial N} < 0$, $\frac{\partial^2 B_l^i}{\partial d_{1l}^i \partial \chi} > 0$ and $\frac{\partial^2 B_l^i}{\partial d_{1l}^i \partial N} < 0$, $\frac{\partial^2 B_l^i}{\partial d_{1l}^i \partial \chi} > 0$. The comparative statics with respect to fiscal capacity follows from the fact that the larger χ is, the smaller $\delta''(B_l)$ is.

Proposition 1.5.3

Proposition 1.5.3: Consider parametrization where the equilibrium is of Type 1 for the constrained Central Planner. (i) If a) $N < \infty$ and $\chi < \chi^*(N)$ or b) if $N \rightarrow \infty$, for any χ , a minimum capital requirement (where $\rho^* = \frac{n}{k_0^{CP}}$) is sufficient to replicate the constrained Central Planner's allocation where $\chi^*(N)$ is pinned down by the system of equations $M(k_{1l}^T) = 0$ and $BC_{1l}(k_{1l}^T) = 0$ where

$$\begin{aligned} M(k_{1l}^T) &= \frac{1}{N} F''(k_{1l}^T) k_{1l}^T + \frac{1}{\frac{\partial^2 \delta(B_l, \chi)}{\partial B_l \partial B_l} N} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} + F'(k_{1l}^T) - 1 = 0 \\ BC_{1l}(k_{1l}^T) &= \pi_l [k_{1l}^T (F'(k_{1l}^T) - \theta(1 - \gamma)) + B_l(k_{1l}^T, \chi)] - \left([1 - \theta(1 - \gamma) + \pi_l(\gamma - a_{1l})] \frac{n}{\rho^*} - n \right) \end{aligned}$$

(ii) if $N < \infty$ and $\chi > \chi^*(N)$, a second instrument is required in the form of a limit on the payment pledged in the crisis state to consumers (where $\nu^* = d_{1l}^{CP}$). Part 2) (i) a) If $\chi > 0$ and $N > N^*(\chi)$ or b) if $\chi = 0$, for any N , a minimum capital requirement (where $\rho^* = \frac{n}{k_0^{CP}}$) is sufficient to replicate the constrained Central Planner's allocation where $N^*(\chi)$ is pinned down by the system of equations $M(k_{1l}^T) = 0$ and $BC_{1l}(k_{1l}^T) = 0$. (ii) If $\chi > 0$ and $N < N^*(\chi)$ a second instrument is required in the form of a limit on the payment pledged in the crisis state to consumers.

Proof of Proposition 1.5.3:

Part 1) First note that in order for the decentralized equilibrium to be of Type 1, it will have to be the case that ρ^* is such that there is a fire-sale in the crisis state and also that the banker does not optimally borrow to the maximum in period zero. The conditions for that are given below.

$$\frac{[1 - \theta(1 - \gamma) + \pi_l(\gamma - a_{1l})]}{(\delta')^{-1} \left(\frac{A - \gamma}{1 - \theta(1 - \gamma)} \right) + n} n > \rho^* > 1 - \pi_h \theta(1 - \gamma) - \theta \pi_l (F'(k_{1l}^T) - \gamma)$$

Both inequalities will be satisfied since I consider only equilibria of Type 1 for the constrained Central Planner (hence the second inequality is satisfied) where I already proved that there is always fire-sale in the crisis state given the assumptions made (hence the first inequality is satisfied). The decentralized equilibrium will be of Type 1 as long as $M(k_{1l}^T) < 0$ where k_{1l}^T is determined by $BC_{1l}(k_{1l}^T, \chi) = 0$ (same as equation A.1.50). If $N \rightarrow \infty$ then $M(k_{1l}^T) < 0$ for every $k_{1l}^T \in [0, k_{1l}^{T, \max}]$ and, hence, the decentralized equilibrium will be always of Type 1. If $N < \infty$, whether the equilibrium is of Type 1 depends on whether k_{1l}^T is such that $M(k_{1l}^T) < 0$. The figure below depicts $M(k_{1l}^T, \chi)$ and the budget constraint in the crisis state, $BC_{1l}(k_{1l}^T, \chi)$ for two different values of fiscal capacity where the equilibrium is of Type 1.⁵

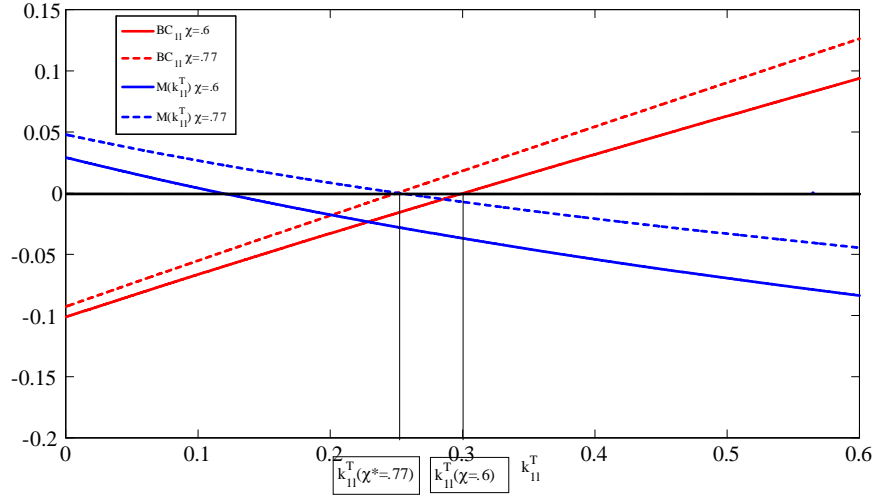


Figure Assumption 1.9

As the fiscal capacity increases, $BC_{1l}(k_{1l}^T)$ shifts up and so does $M(k_{1l}^T)$. Therefore, to solve for the maximum possible fiscal capacity such that Type 1 equilibrium is achieved one has to solve the following system of two equations and two unknowns, k_{1l}^T and χ^* , for a given N .

⁵The parameters are the same as in Figure 1.5 (where $N=3$).

$$\begin{aligned} M(k_{1l}^T) &= 0 \\ BC_{1l}(k_{1l}^T) &= 0 \end{aligned}$$

If $\chi > \chi^*(N)$ then B_{1l} will cross the zero line at a point that is to the left of $M(k_{1l}^T)$ and hence the requirement $M(k_{1l}^T) < 0$ will be violated and the equilibrium will not be of Type 1.

Part 2) If $\left[F''(k_{1l}^T) k_{1l}^T + \frac{1}{\frac{\partial^2 \delta(B_l, \chi)}{\partial B_l \partial B_l}} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} \right] > 0$ then

$$\frac{\partial M(k_{1l}^T)}{\partial N} = -\frac{1}{N^2} \left[F''(k_{1l}^T) k_{1l}^T + \frac{1}{\frac{\partial^2 \delta(B_l, \chi)}{\partial B_l \partial B_l}} \frac{\partial z_{1l}^{1,P}}{\partial k_{1l}^T} \right] < 0$$

while $BC_{1l}(k_{1l}^T)$ is not a function of N . Therefore, the smaller N is the more $M(k_{1l}^T)$ moves up and the minimum $N^*(\chi)$ as a function of χ for which the equilibrium is of Type 1 is given by solving the system of equations, for a given χ (if $M(k_{1l}^T) < 0$ for every k_{1l}^T given χ then $N^*(\chi) = 1$)

$$\begin{aligned} M(k_{1l}^T) &= 0 \\ BC_{1l}(k_{1l}^T) &= 0 \end{aligned}$$

Proposition 1.6.1

Proposition 1.6.1: *Conditional on Assumptions 1.1-1.8 and 1.10 and the assumptions made on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, if the policy maker has an access to a sufficient number of instruments to replicate the constrained Central Planner's allocation and the parametrization is such that the Central Planner's equilibrium is of Type 1 (Assumption 1.9 is satisfied), the optimal minimum bank capital ratio is higher for more fiscally constrained countries, $\frac{\partial \rho^*}{\partial \chi} < 0$.*

Proof of Proposition 1.6.1: Consider an interior equilibrium for the Central Planner. By setting $\rho^* = \frac{n}{k_0^{CP}}$ and $v^* = d_{1l}^{CP}$ (if necessary) the policy maker can replicate the constrained Central Planner's allocation. Since I proved that $\frac{\partial \psi^{CP}(k_0; \chi)}{\partial k_0} > 0$, it is sufficient to prove that holding k_0 constant, $\frac{\partial \psi^{CP}(\chi; k_0)}{\partial \chi} < 0$ (partial derivative) in order to prove that $\frac{\partial \rho^*}{\partial \chi} > 0$. Define

$$H(\chi; k_0) = \pi_l B_l(k_{1l}^T, \chi) + n + [F'(k_{1l}^T) - \theta(1 - \gamma)] k_{1l}^T \pi_l \quad (\text{A.1.56})$$

$$-k_0(1 - \pi_h \theta(1 - \gamma) + (\gamma - a_{1l} - \theta(1 - \gamma)) \pi_l) = 0 \quad (\text{A.1.57})$$

For a given k_0 , totally differentiate $H(\chi; k_0) = 0$ with respect to χ (partial derivative)

$$\frac{\partial k_{1l}^T(\chi; k_0)}{\partial \chi} = -\frac{\partial B_l(\chi; k_{1l}^T)}{\partial \chi} \frac{1}{\left[\frac{\partial B_l(k_{1l}^T; \chi)}{\partial k_{1l}^T} + F'(k_{1l}^T) - \theta(1 - \gamma) + F''(k_{1l}^T) k_{1l}^T \right]} < 0 \quad (\text{A.1.58})$$

where $\frac{\partial B_l(\chi; k_{1l}^T)}{\partial \chi} > 0$ is the partial derivative of B_l with respect to χ holding k_{1l}^T constant. I can solve for $\frac{\partial B_l(\chi; k_{1l}^T)}{\partial \chi}$ by totally differentiating the first order condition that pins down B_l , holding k_{1l}^T constant. The derivative is given by $0 = \frac{\partial^2 \delta(B_l)}{\partial B_l \partial B_l} \frac{\partial B_l(\chi; k_{1l}^T)}{\partial \chi} + \frac{\partial^2 \delta(B_l)}{\partial B_l \partial \chi} = \eta(\eta - 1) \frac{1}{\chi} B_l^{\eta-2} \frac{\partial B_l(\chi; k_{1l}^T)}{\partial \chi} - \frac{1}{\chi^2} \eta B_l^{\eta-1}$ where $\frac{\partial^2 \delta(B_l)}{\partial B_l \partial \chi}$ is a partial derivative holding B_l constant. Since $\frac{\partial^2 \delta(B_l)}{\partial B_l \partial \chi} < 0$ and $\frac{\partial^2 \delta(B_l)}{\partial B_l \partial B_l} > 0$, $\frac{\partial B_l(\chi; k_{1l}^T)}{\partial \chi} = -\frac{\partial^2 \delta(B_l)}{\partial B_l \partial \chi} / \frac{\partial^2 \delta(B_l)}{\partial B_l \partial B_l} = \frac{B_l}{(\eta-1)\chi} > 0$.

Also for a given χ as shown in Lemma A.1.1 $\frac{\partial B_l(k_{1l}^T; \chi)}{\partial k_{1l}^T} > 0$. The fact that $\frac{\partial k_{1l}^T(\chi; k_0)}{\partial \chi} < 0$ is intuitive and means that for a given level period zero investment, the fire sale will be larger for the country with the smaller fiscal capacity.

$$\frac{\partial \psi^{CP}(\chi; k_0)}{\partial \chi} = \frac{\partial z_{1l}^{CP}}{\partial \chi} - \frac{\partial z_0^{CP}}{\partial \chi} = \left[\frac{1 - \theta(1 - \gamma) + \pi_l(\gamma - a_{1l})}{[1 - \pi_h \theta(1 - \gamma)]} \right] \frac{\partial k_{1l}^T(\chi; k_0)}{\partial \chi} \frac{\partial z_{1l}^{CP}}{\partial k_{1l}^T} < 0$$

where I proved that $\frac{\partial z_{1l}^{CP}}{\partial k_{1l}^T} > 0$ in Lemma A.1.1 since $z_{1l}^{1,P} = z_{1l}^{CP}$ if the equilibrium is of Type 1. This completes the proof.

Proposition 1.7.1

Proposition 1.7.1: *Conditional on Assumptions 1.1-1.10 and on the functional forms of $F(\cdot)$ and $\delta(\cdot)$, if the policy maker has an access to two ex-ante instruments — an ex-ante tax on period zero investment ("price" instrument), τ_{k_0} , and a limit on the payment promised in the crisis state, v , one can show that $\tau_{k_0} > 0$. If $N \rightarrow \infty$ (no moral hazard) then $\frac{\partial \tau_{k_0}}{\partial \chi} = 0$. If $1 < N < \infty$ then $\frac{\partial \tau_{k_0}}{\partial \chi} > 0$. τ_{k_0} and $\frac{\partial \tau_{k_0}}{\partial \chi}$ are given by*

$$\tau_{k_0} = \left[\frac{z_{1l}^{CP}(k_{1l}^T)}{z_{1l}^*(k_{1l}^T, \chi)} - 1 \right] \Phi > 0$$

$$\frac{\partial \tau_{k_0}}{\partial \chi} = - \frac{\partial z_{1l}^* (k_{1l}^T, \chi)}{\partial \chi} \frac{z_{1l}^{CP} (k_{1l}^T, \chi)}{[z_{1l}^* (k_{1l}^T, \chi)]^2} \Phi \geq 0$$

where $\Phi = \frac{[1-\theta(1-\gamma)+\pi_l(\gamma-a_{1l})]}{(1-\frac{1}{N})} > 0$.

Proof of Proposition 1.7.1: Conditional on the policy maker having an access to two ex-ante instruments — an ex-ante tax on period zero investment ("price" instrument), $\tau_{k_0}^i$, and a limit on the payment promised in the crisis state, $d_{1l}^i \leq v^i$, the constrained Central Planner's allocation can be replicated. Consider parametrization such as the equilibrium is of Type 1 for the constrained Central Planner. Given that the policy maker has sufficient instruments to replicate the constrained Central Planner's allocation, the only Type of equilibrium to consider is of Type 1.

The only difference between the problem of the banker where the ex-ante instrument is a "price" instrument and the problem of the banker where the ex-ante instrument is a "quantity" instrument is that the period zero budget constraints becomes

$$k_0^i (1 + \tau_{k_0}^i) - n + T_{k_0}^i \leq \sum_s \pi_s d_{1s}^i \quad [z_0^i] \quad (\text{A.1.59})$$

where $\tau_{k_0}^i$ is the tax on period zero capital. The revenues from the proportional tax are distributed equally back to the bankers using the lump sum tax, $T_{k_0}^i = -\sum_{i=1}^N \frac{1}{N} k_0^i \tau_{k_0}^i$. Banker i chooses k_0^i at the end of $t = 0$, while $\tau_{k_0}^i$ is determined in the beginning of $t = 0$ and banker i takes it as given. However, banker i internalizes the fact that he affects the lump sum tax $T_{k_0}^i$. Banker's i optimization problem at the end of $t = 0$ is

$$\max_{k_0^i, d_{1s}^i, k_{1s}^i} \sum_s \pi_s (A + (1 - \theta) (1 - \gamma)) k_{1s}^i$$

subject to the period one budget constraint, the borrowing constraint

$$k_{1s}^i (F' (k_{1s}^T) - \theta (1 - \gamma)) + d_{1s}^i \leq (F' (k_{1s}^T) + a_{1s} - \gamma) k_0^i + B_s^i \quad [\pi_s z_{1s}^{i,0}] \quad (\text{A.1.60})$$

$$d_{1s}^i \leq \theta (F' (k_{1s}^T) - \gamma) k_0^i \quad [\pi_s \lambda_{1s}^i]$$

and to the period zero budget constraint given by equation A.1.59. First order condition with respect to k_0^i

$$\begin{aligned} & \sum_s \pi_s z_{1s}^{i,0} \left(F' (k_{1s}^T) + a_{1s} - \gamma + \frac{1}{N} F'' (k_{1s}^T) k_{1s}^{i,T} + \frac{\partial B_s^i}{\partial k_0^i} \right) \\ & - z_0^i \left[1 + \tau_{k_0}^i \left(1 - \frac{1}{N} \right) \right] + \sum_s \pi_s \lambda_{1s}^i \theta \left(F' (k_{1s}^T) - \gamma + \frac{1}{N} F'' (k_{1s}^T) k_0^i \right) = 0 \end{aligned} \quad (\text{A.1.61})$$

and the rest of the first order conditions are the same as in the decentralized equilibrium with a "quantity" ex-ante instrument. After imposing a symmetric equilibrium and using the fact that $z_0 = z_{1l} > z_{1h}$, $\lambda_{2s} > 0, \lambda_{1l} = 0$ and $\lambda_{1h} > 0$, one can re-write the first order condition with respect to k_0 from the decentralized problem as

$$z_{1l}^* \left[1 + \tau_{k_0} \left(1 - \frac{1}{N} \right) - \pi_h \theta (1 - \gamma) - \pi_l (\theta (1 - \gamma) + a_{1l} - \gamma) \right] \quad (\text{A.1.62})$$

$$= \frac{(A + (1 - \theta)(1 - \gamma))}{1 - \theta(1 - \gamma)} (1 - \theta(1 - \gamma) + \pi_h (a_{1h} - \gamma)) \quad (\text{A.1.63})$$

where z_{1l}^* is given by equation A.1.29. From equation A.1.44

$$z_{1l}^{CP} (1 - \pi_h \theta [1 - \gamma] - \pi_l (\theta (1 - \gamma) + a_{1l} - \gamma)) = \frac{(A + (1 - \theta)(1 - \gamma))}{1 - \theta(1 - \gamma)} [\pi_h (a_{1h} - \gamma) + 1 - \theta(1 - \gamma)] \quad (\text{A.1.64})$$

Subtracting equation A.1.62 from equation A.1.64, since $z_{1l}^{CP} (k_{1l}^T) - z_{1l}^* (k_{1l}^T, \chi) > 0$, which I proved in Corollary 1.4.1,

$$\tau_{k_0} = \left[\frac{z_{1l}^{CP} (k_{1l}^T)}{z_{1l}^* (k_{1l}^T, \chi)} - 1 \right] \Phi > 0$$

where

$$\Phi = \frac{[1 - \theta(1 - \gamma) + \pi_l (\gamma - a_{1l})]}{(1 - \frac{1}{N})} > 0.$$

Since the equilibrium k_{1l}^T does not vary with χ in the Central Planner problem and z_{1l}^{CP} is a function only of k_{1l}^T , $\frac{\partial z_{1l}^{CP}(k_{1l}^T)}{\partial \chi} = 0$

$$\frac{\partial \tau_{k_0}}{\partial \chi} = - \frac{\partial z_{1l}^* (k_{1l}^T, \chi)}{\partial \chi} \frac{z_{1l}^{CP} (k_{1l}^T, \chi)}{[z_{1l}^{*,N} (k_{1l}^T, \chi)]^2} \Phi \geq 0$$

where

$$\frac{\partial z_{1l}^* (k_{1l}^T, \chi)}{\partial \chi} = -\frac{\partial B_l}{\partial k_0^i \partial \chi} \frac{z_{1l}^* (k_{1l}^T, \chi)}{\left[F' (k_{1l}^T) - \theta (1 - \gamma) + \frac{1}{N} F'' (k_{1l}^T) k_{1l}^T + \frac{\partial B_l}{\partial k_0^i} \right]} \leq 0$$

where in Corollary 1.5.2 I proved $\frac{\partial B_l}{\partial k_0^i \partial \chi} \geq 0$. Also I take into account that the equilibrium k_{1l}^T is not a function of χ since it's determined by the first order condition of the Central Planner.

If $N \rightarrow \infty$, z_{1l}^* is not a function of χ because $\frac{\partial B_l}{\partial k_0^i \partial \chi} = 0$ which implies $\frac{\partial \tau_{k_0}}{\partial \chi} = 0$. If $1 < N < \infty$, $\frac{\partial B_l}{\partial k_0^i \partial \chi} > 0$ and $\frac{\partial \tau_{k_0}}{\partial \chi} > 0$.

Appendix B

Supplement to Chapter 2

B.1 Appendix

B.1.1 The Problem of the Entrepreneur

I solve the problem of the entrepreneur backwards. In $t = 1$, all the uncertainty is resolved. The entrepreneur takes $\bar{R}_1^l(s_1)$, $D_2(s_1)$ and $D_1(s_1)$ as given and optimizes

$$\begin{aligned} \max_{L_1(s_1)} & \left[\bar{A}(L_1(s_1))^\alpha - \bar{R}_1^l(s_1)L_1(s_1) + D_2(s_1) \right] \\ & + A_1 L_0^\alpha - R_1^l(s_1)L_0 + D_1(s_1) \end{aligned}$$

The first order condition with respect to $L_1(s_1)$ determines the demand for aggregate loans as a function of $\bar{R}_1^l(s_1)$

$$L_1(s_1) = \left[\frac{\bar{R}_1^l(s_1)}{\alpha \bar{A}} \right]^{\frac{1}{\alpha-1}} \quad (\text{B.1.1})$$

Alternatively the problem can be re-written as (in order to solve for $L_{i,1}$)

$$\max_{L_{i,1}} \bar{A} \left(\left[\sum_{i=1}^n \frac{1}{n} (L_{i,1})^{(\rho-1)/\rho} \right]^{\frac{\rho}{\rho-1}} \right)^\alpha - \sum_{i=1}^n \frac{1}{n} \bar{R}_{i,1}^l L_{i,1}$$

The first order condition with respect to $L_{i,1}$ is given by

$$L_{i,1} = \left[\frac{\bar{R}_{i,1}^l}{(L_1)^{\alpha-1+\frac{1}{\rho}} \alpha \bar{A}} \right]^{-\rho} = L_1 \left[\frac{\bar{R}_{i,1}^l}{\bar{R}_1^l} \right]^{-\rho} \quad (\text{B.1.2})$$

where the aggregate lending rate is $\bar{R}_1^l = \left[\sum_{i=1}^n \frac{1}{n} (\bar{R}_{i,1}^l)^{(1-\rho)} \right]^{\frac{1}{(1-\rho)}}$. In $t = 0$ the optimization problem simplifies to

$$\max_{L_{i,0}} \pi_H \left[A_H \left(\left[\sum_{i=1}^n \frac{1}{n} (L_{i,0})^{(\rho-1)/\rho} \right]^{\frac{\rho}{\rho-1}} \right)^\alpha - \sum_{i=1}^n \frac{1}{n} \bar{R}_{i,0}^l L_{i,0} \right]$$

After taking first order conditions and re-arranging, the equilibrium system of period zero equations is

$$\begin{aligned} L_0 &= \left[\frac{\bar{R}_0^l}{\alpha A_H} \right]^{\frac{1}{\alpha-1}} \\ L_{i,0} &= \left[\frac{\bar{R}_{i,0}^l}{(L_0)^{\alpha-1+\frac{1}{\rho}} \alpha A_H} \right]^{-\rho} = L_0 \left[\frac{\bar{R}_{i,0}^l}{\bar{R}_0^l} \right]^{-\rho} \\ \bar{R}_0^l &= \left[\sum_{i=1}^n \frac{1}{n} (\bar{R}_{i,0}^l)^{(1-\rho)} \right]^{\frac{1}{(1-\rho)}} \end{aligned} \quad (\text{B.1.3})$$

B.1.2 The Problem of Banker i

I solve the problem of banker i backwards. Since all the uncertainty is resolved in the middle period $t = 1$, there is no default in $t = 2$. All the equations are a function of the state s_1 (for now the notation is suppressed). The banker maximizes

$$\max_{L_{i,1}} \bar{R}_{i,1}^l (L_{i,1}) L_{i,1} - R_1^f [L_{i,1} - N_{i,1}] + \lambda_{i,1} [N_{i,1} - \eta L_{i,1}]$$

The first order condition with respect to $L_{i,1}$ is

$$\left[\underbrace{\frac{\partial \bar{R}_{i,1}^l (L_{i,1})}{\partial L_{i,1}} L_{i,1}}_{\text{marginal impact on } R_1^l} + \bar{R}_{i,1}^l - R_1^f \right] - \lambda_{i,1} \eta = 0 \quad (\text{B.1.4})$$

where $\frac{\partial \bar{R}_{i,1}^l (L_{i,1})}{\partial L_{i,1}}$ is given by totally differentiating the rewritten equation B.1.2

$$\bar{R}_{i,1}^l (L_{i,1}) = (L_{i,1})^{-\frac{1}{\rho}} (L_1)^{\alpha-1+\frac{1}{\rho}} \alpha \bar{A}$$

with respect to $L_{i,1}$ and taking into account the fact $\frac{\partial L_t}{\partial L_{i,t}} = \frac{1}{n} \left(\frac{L_t}{L_{i,t}} \right)^{\frac{1}{\rho}}$.

$$\frac{\partial \bar{R}_{i,1}^l}{\partial L_{i,1}} = - \left[\left[\frac{\bar{R}_{i,1}^l}{\bar{R}_1^l} \right]^\rho - ((\alpha - 1)\rho + 1) \frac{1}{n} \left(\frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}} \right] \left(\frac{\bar{R}_{i,1}^l}{L_1} \right) \frac{1}{\rho} \quad (\text{B.1.5})$$

If the net worth constraint does not bind in $t = 1$ ($\lambda_{i,1} = 0$), one can re-write equation B.1.4 as (after plugging in equation B.1.5)

$$L_{i,1} \left[\frac{(\bar{R}_1^l)^{-\rho}}{(\bar{R}_{i,1}^l)^{-\rho}} - (\rho(\alpha - 1) + 1) \frac{1}{n} \left(\frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}} \right] \frac{1}{\rho} \frac{\bar{R}_{i,1}^l}{L_1} = \bar{R}_{i,1}^l - R_1^f$$

Notice that if the net worth constraint is not binding, the first order condition is not a function of period zero variables. Since banks are symmetric, I consider the symmetric equilibrium which implies $\bar{R}_{i,1}^l = \bar{R}_1^l$ and $L_{i,1} = L_1$ and the first order condition simplifies to

$$\bar{R}_1^l = \gamma R_1^f$$

where $\gamma = \frac{1}{(1 - \frac{1}{\rho})(1 - \frac{1}{n}) - (1 - \alpha)\frac{1}{n}}$ is the mark-up.

If the net worth constraint binds in $t = 1$ ($\lambda_{i,1} > 0$), then the amount of loans in period one becomes

$$L_{i,1} = \frac{1}{\eta} N_{i,1}$$

where

$$\lambda_{i,1} = \frac{1}{\eta} \left[\bar{R}_{i,1}^l - R_1^f - L_{i,1} \left[\frac{(\bar{R}_1^l)^{-\rho}}{(\bar{R}_{i,1}^l)^{-\rho}} - (\rho(\alpha - 1) + 1) \frac{1}{n} \left(\frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}} \right] \frac{1}{\rho} \frac{\bar{R}_{i,1}^l}{L_1} \right]$$

$$N_{i,1}(s_L) = A_L(L_0)^\alpha \frac{L_{i,0}}{\hat{L}_0} - R_0^f [L_{i,0} - N_{i,0}]$$

$$N_{i,1}(s_H) = \bar{R}_{i,0} L_{i,0} - R_0^f [L_{i,0} - N_{i,0}]$$

Next I solve the optimization problem of banker i in $t = 0$. Assuming no default by banker i in $t = 1$, which in equilibrium will be true, and that the net worth constraint binds in the low state in $t = 1$ and does not bind in the high state in $t = 1$, banker i maximizes the expected dividend payment in the last period (it

is never optimal to pay dividends before $t = 2$).

$$\begin{aligned}
& \max_{L_{i,0}} E_0 N_{i,2} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}] \\
&= \max_{\bar{R}_{i,0}^l} \pi_H \left[\bar{R}_{i,1}^l(L_{i,0}; s_H) - R_1^f \right] L_{i,1}(s_H) + (1 - \pi_H) \left[\bar{R}_{i,1}^l(L_{i,0}; s_L) - R_1^f \right] L_{i,1}(s_L) \\
& \quad + R_1^f \left(\pi_H \bar{R}_{i,0}^l(L_{i,0}) + (1 - \pi_H) A_L L_0^\alpha \frac{1}{\hat{L}_0} - R_0^f \right) L_{i,0} + R_1^f R_0^f N_{i,0} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}]
\end{aligned}$$

Since in $t = 1$ the problem is static in the states of nature where the net worth constraint is not binding and I consider parametrization where it is not binding in the high state in $t = 1$, $L_{i,0}$ will not affect $L_{i,1}(s_H)$ and $R_{i,1}(s_H)$. With that in mind, the first order condition with respect to $L_{i,0}$ becomes

$$\begin{aligned}
& (1 - \pi_H) \left[\frac{\partial \bar{R}_{i,1}^l(s_L)}{\partial L_{i,0}} L_{i,1}(s_L) + \left(\bar{R}_{i,1}^l(s_L) - R_{i,1}^f \right) \frac{\partial L_{i,1}(s_L)}{\partial L_{i,0}} \right] + \\
& \quad R_{i,1}^f \left(\pi_H \bar{R}_{i,0}^l + (1 - \pi_H) A_L L_0^\alpha \frac{1}{\hat{L}_0} - R_{i,0}^f \right) + \\
& \quad R_{i,1}^f \left(\pi_H \frac{\partial \bar{R}_{i,0}^l}{\partial L_{i,0}} + (1 - \pi_H) A_L \left[\alpha L_0^{\alpha-1} \frac{\partial L_0}{\partial L_{i,0}} \frac{1}{\hat{L}_0} - L_0^\alpha \hat{L}_0^{-2} \frac{1}{n} \right] \right) L_{i,0} - \lambda_{i,0} \eta = 0
\end{aligned}$$

where

$$\frac{\partial \bar{R}_{i,1}^l(s_L)}{\partial L_{i,0}} = - \left[\left[\frac{\bar{R}_{i,1}^l(s_L)}{\bar{R}_1^l(s_L)} \right]^\rho - ((\alpha - 1)\rho + 1) \frac{1}{n} \left(\frac{L_1(s_L)}{L_{i,1}(s_L)} \right)^{\frac{1}{\rho}} \right] \left(\frac{\bar{R}_{i,1}^l(s_L)}{L_1(s_L)} \right) \frac{1}{\rho} \frac{1}{\eta} \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}}$$

$$\frac{\partial L_{i,1}(s_L)}{\partial L_{i,0}} = \frac{1}{\eta} \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} \tag{B.1.6}$$

$$\frac{\partial \bar{R}_{i,0}^l}{\partial L_{i,0}} = - \left[\left[\frac{\bar{R}_{i,0}^l}{\bar{R}_0^l} \right]^\rho - ((\alpha - 1)\rho + 1) \frac{1}{n} \left(\frac{L_0}{L_{i,0}} \right)^{\frac{1}{\rho}} \right] \left(\frac{\bar{R}_{i,0}^l}{L_0} \right) \frac{1}{\rho} \tag{B.1.7}$$

$$\frac{\partial L_t}{\partial L_{i,t}} = \frac{1}{n} \left(\frac{L_t}{L_{i,t}} \right)^{\frac{1}{\rho}}$$

$$\frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = A_L L_0^\alpha \frac{1}{\hat{L}_0} + A_L \frac{1}{n} \left(\frac{L_0}{L_{i,0}} \right)^{\frac{1}{\rho}} \alpha (L_0)^{\alpha-1} \frac{L_{i,0}}{\hat{L}_0} - A_L L_0^\alpha \frac{L_{i,0}}{\hat{L}_0^2} \frac{1}{n} - R_0^f$$

$$N_{i,1}(s_L) = A_L(L_0)^\alpha \frac{L_{i,0}}{\hat{L}_0} - R_0^f [L_{i,0} - N_{i,0}]$$

$$N_{i,1}(s_H) = \bar{R}_{i,0}^l L_{i,0} - R_0^f [L_{i,0} - N_{i,0}]$$

Assume that we start from normal times where the borrowing constraint doesn't bind in $t = 0$, $\lambda_{i,0} = 0$ and that the equilibrium is symmetric. One can re-write the first order condition with respect to $L_{i,0}$ as

$$MC(L_0) = -(1 - \pi_H) \lambda_1(s_L) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = R_1^f E_0 \frac{\partial N_{i,1}(s_1)}{\partial L_{i,0}} = MB(L_0) \quad (\text{B.1.8})$$

where

$$\frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = A_L L_0^{\alpha-1} \left(1 - \frac{1}{n} (1 - \alpha) \right) - R_0^f < 0$$

$$\frac{\partial N_{i,1}(s_H)}{\partial L_{i,0}} = \alpha A_H L_0^{\alpha-1} \frac{1}{\gamma} - R_0^f > 0$$

and if $\lambda_1(s_L) > 0$,

$$\lambda_1(s_L) = \frac{1}{\eta} \frac{\partial N_{i,2}(s_L)}{\partial L_{i,1}(s_L)} = \frac{1}{\eta} \left[\alpha \bar{A}(L_1(s_L))^{\alpha-1} \frac{1}{\gamma} - R_1^f \right] > 0 \quad (\text{B.1.9})$$

Notice that the bank will never choose an allocation that leads to bank bankruptcy in $t = 1$ because, given the assumptions made in this model, no bank loans would imply no investment and $K_1 = L_1 = 0$. As a result, if $L_1 \rightarrow 0$, then $\lambda_1(s_L) \rightarrow \infty$ and $MC(L_0) \rightarrow \infty$ and, therefore, banker i will never choose period zero allocation which will lead to bank default. If the entrepreneur has another source of income, then this result can be changed and one can generalize the model to include bank default. After plugging in all the equations for the case $\lambda_1(s_L) > 0$,

$$\begin{aligned} (1 - \pi_H) \frac{1}{\eta} \left[\left(1 - (1 - \alpha) \frac{1}{n} \right) A_L L_0^{\alpha-1} - R_0^f \right] \left[\alpha \bar{A}(L_1(s_L))^{\alpha-1} \frac{1}{\gamma} - R_1^f \right] + \\ R_1^f \left(\pi_H(L_0)^{\alpha-1} \alpha A_H \frac{1}{\gamma} + (1 - \pi_H) A_L L_0^{\alpha-1} \left(1 - (1 - \alpha) \frac{1}{n} \right) - R_0^f \right) = 0 \end{aligned} \quad (\text{B.1.10})$$

B.1.3 Constrained Central Planner's Problem

The Central Planner (CP) chooses the amount of loans provided by every banker, taking into account that the equilibrium played is symmetric. The CP also takes into account the net worth constraint and the first

order conditions of the entrepreneur. He internalizes his impact on the return of the bankers. The period one optimization problem of the CP is

$$\max_{L_1} \bar{A}L_1^\alpha - R_1^f [L_1 - N_1] + \lambda_1^{CP} [N_1 - \eta L_1]$$

The first order condition with respect to L_1 is given by

$$\alpha \bar{A}L_1^{\alpha-1} - R_1^f - \lambda_1^{CP} \eta = 0.$$

If the net worth constraint does not bind ($\lambda_1^{CP} = 0$), the bank will lend to the entrepreneur up to the point where the marginal cost equals the marginal benefit from the Central Planner's point of view, $\alpha \bar{A}L_1^{\alpha-1} = R_1^f$. If the net worth constraint binds ($\lambda_1^{CP} > 0$), then

$$L_1(s_L) = \frac{1}{\eta} N_1(s_L)$$

The optimization problem at $t = 0$ is given by

$$\begin{aligned} & \max_{L_0} (1 - \pi_H) (\bar{A}(L_1(s_L))^\alpha - \bar{R}_1^l(s_L) L_1(s_L)) \\ & + \pi_H (R_1^f (A_H(L_0)^\alpha - \bar{R}_0^l(L_0) L_0) + \bar{A}(L_1(s_H))^\alpha - \bar{R}_1^l(s_H) L_1(s_H)) \\ & + \pi_H [\bar{R}_1^l(s_H) - R_1^f] L_1(s_H) + (1 - \pi_H) [\bar{R}_1^l(s_L) - R_1^f] L_1(s_L) \\ & + R_1^f (\pi_H \bar{R}_0^l + (1 - \pi_H) A_L L_0^{\alpha-1} - R_0^f) L_0 + R_1^f R_0^f N_0 + \lambda_0 [N_0 - \eta L_0] \end{aligned}$$

Re-writing the optimization problem, after plugging in for the first order conditions of the entrepreneur and simplifying,

$$\begin{aligned} & \max_{L_0} (1 - \pi_H) (\bar{A}(L_1(s_L))^\alpha - R_1^f L_1(s_L)) + \pi_H (\bar{A}(L_1(s_H))^\alpha - R_1^f L_1(s_H)) \\ & + R_1^f E_0 A_1 L_0^\alpha - R_1^f R_0^f L_0 + R_1^f R_0^f N_0 + \lambda_0 [N_0 - \eta L_0] \end{aligned}$$

Since $L_1(s_H)$ is not a function of L_0 , the first order condition with respect to L_0 is given by

$$(1 - \pi_H) [\alpha \bar{A}(L_1(s_L))^{\alpha-1} - R_1^f] \frac{\partial L_1(s_L)}{\partial L_0} + R_1^f (E_0 A_1 \alpha (L_0)^{\alpha-1} - R_0^f) - \lambda_0 \eta = 0$$

where

$$\frac{\partial L_1(s_L)}{\partial L_0} = \frac{1}{\eta} \frac{\partial N_1(s_L)}{\partial L_0} = \frac{1}{\eta} \left(\alpha A_L(L_0)^{\alpha-1} - R_0^f \right)$$

Combining all the equations, one gets

$$MC^{CP}(L_0) = -(1 - \pi_H) \frac{1}{\eta} \left(\alpha A_L L_0^{\alpha-1} - R_0^f \right) \left[\alpha \bar{A}(L_1(s_L))^{\alpha-1} - R_1^f \right] \quad (\text{B.1.11})$$

$$= R_1^f \left(\alpha E_1 A_1 L_0^{\alpha-1} - R_0^f \right) = MB^{CP}(L_0) \quad (\text{B.1.12})$$

Next I proof existence and uniqueness conditional on the equilibrium being such that $\lambda_0^{CP} = 0$, $\lambda_1^{CP}(s_L) > 0$ and $\lambda_1^{CP}(s_H) = 0$. First, I prove that $MB^{CP'}(L_0) - MC^{CP'}(L_0) < 0$.

$$MB^{CP'}(L_0) = R_1^f (\alpha - 1) \alpha E_0 A_1 L_0^{\alpha-2} < 0$$

$$MC^{CP'}(L_0) = (1 - \pi_H) (1 - \alpha) \left\{ \begin{aligned} & \alpha A_L L_0^{\alpha-2} \frac{1}{\eta} \left[\alpha \bar{A}(L_1(s_L))^{\alpha-1} - R_1^f \right] \\ & + \left[\frac{1}{\eta} \left(\alpha A_L L_0^{\alpha-1} - R_0^f \right) \right]^2 \alpha \bar{A}(L_1(s_L))^{\alpha-2} \end{aligned} \right\} > 0$$

Since also $\lim_{L_0 \rightarrow 0} [MB^{CP}(L_0) - MC^{CP}(L_0)] \rightarrow \infty$ and $\lim_{L_0 \rightarrow \infty} [MB^{CP}(L_0) - MC^{CP}(L_0)] \rightarrow -\infty$, this is sufficient to prove existence and uniqueness.

B.1.4 Ramsey Problem

First, I re-derive the problem of the entrepreneur given the presence of subsidies. As before, I solve the model backwards. In $t = 1$, all the uncertainty is resolved and the optimization problem of the entrepreneur after plugging in the budget constraint simplifies to

$$\max_{L_1} \left[\bar{A}(L_1(s_1))^\alpha - \tilde{R}_1^l(s_1) L_1(s_1) \right]$$

where $\tilde{R}_1^l(s_1) L_1(s_1) = \sum_{i=1}^n \frac{1}{n} (1 - \tau_{i,1}^s) \bar{R}_{i,1}^l L_{i,1}$. Notice that the subsidy is at the individual bank interest rate level and the aggregate interest rate, \tilde{R}_1^l , is net of subsidies. The first order condition with respect to L_t determines the demand for aggregate loans as a function of $L_1(s_1)$

$$L_1(s_1) = \left[\frac{\tilde{R}_1^l(s_1)}{\alpha \bar{A}} \right]^{\frac{1}{\alpha-1}} \quad (\text{B.1.13})$$

Alternatively the problem can be re-written as

$$\max_{L_{i,1}} \bar{A} \left(\left[\sum_{i=1}^n \frac{1}{n} (L_{i,1})^{(\rho-1)/\rho} \right]^{\frac{\rho}{\rho-1}} \right)^\alpha - \sum_{i=1}^n \frac{1}{n} (1 - \tau_{i,1}^s) \bar{R}_{i,1}^l L_{i,1}$$

The first order condition with respect to $L_{i,t}$ is given by

$$L_{i,1} = \left[\frac{(1 - \tau_{i,1}^s) \bar{R}_{i,1}^l}{(L_1)^{\alpha-1+\frac{1}{\rho}} \alpha \bar{A}} \right]^{-\rho} = L_1 \left[\frac{(1 - \tau_{i,1}^s) \bar{R}_{i,1}^l}{\tilde{R}_1^l} \right]^{-\rho}$$

Also the aggregate lending rate is

$$\tilde{R}_1^l = \left[\sum_{i=1}^n \frac{1}{n} ((1 - \tau_{i,1}^s) \bar{R}_{i,1}^l)^{(1-\rho)} \right]^{\frac{1}{(1-\rho)}} \quad (\text{B.1.14})$$

In $t = 0$, given that I consider parametrization where the firm defaults in the crisis state, the optimization problem is

$$\max_{L_{i,0}} \pi_H \left[A_H \left(\left[\sum_{i=1}^n \frac{1}{n} (L_{i,0})^{(\rho-1)/\rho} \right]^{\frac{\rho}{\rho-1}} \right)^\alpha - \sum_{i=1}^n \frac{1}{n} (1 - \tau_{i,0}^s) \bar{R}_{i,0}^l L_{i,0} \right]$$

After taking the first order conditions and re-arranging the equilibrium system of equations is

$$L_0 = \left[\frac{\tilde{R}_0^l}{\alpha A_H} \right]^{\frac{1}{\alpha-1}}$$

$$L_{i,0} = \left[\frac{(1 - \tau_{i,0}^s) \bar{R}_{i,0}^l}{(L_0)^{\alpha-1+\frac{1}{\rho}} \alpha A_H} \right]^{-\rho} = L_0 \left[\frac{(1 - \tau_{i,0}^s) \bar{R}_{i,0}^l}{\tilde{R}_0^l} \right]^{-\rho} \quad (\text{B.1.15})$$

$$\tilde{R}_0^l = \left[\sum_{i=1}^n \frac{1}{n} ((1 - \tau_{i,0}^s) \bar{R}_{i,0}^l)^{(1-\rho)} \right]^{\frac{1}{(1-\rho)}} \quad (\text{B.1.16})$$

Next I re-derive the banker's problem taking into account the policy instruments. Since all the uncertainty is resolved in the middle period, $t = 1$, there is no firm default in $t = 2$. All the equations are a function of the state s_1 (for now the notation is suppressed). The banker maximizes his period two dividend payments/profits

$$\max_{L_{i,1}} \bar{R}_{i,1}^l (L_{i,1}) L_{i,1} - (1 + \tau_{i,1}^{cc}) R_1^f [L_{i,1} - N_{i,1}] + T_{i,2} + \lambda_{i,1} [N_{i,1} - \eta L_{i,1}]$$

where $T_{i,2} = \tau_{i,1}^{cc} R_1^f [L_{i,1} - N_{i,1}] - \tau_{i,t}^s \bar{R}_{i,t}^l L_{i,1}$ is predetermined in the beginning of period zero since I assumed that the policy maker is able to commit. Therefore, banker i takes $T_{i,2}$ as given. The policy maker also anticipates the optimal actions of the bankers. The first order condition with respect to $L_{i,1}$ is

$$\left[\underbrace{\frac{\partial \bar{R}_{i,1}^l(L_{i,1})}{\partial L_{i,1}} L_{i,1}}_{\text{marginal impact on } R_1^l} + \bar{R}_{i,1}^l - (1 + \tau_{i,1}^{cc}) R_1^f \right] - \lambda_{i,1} \eta = 0$$

where $\frac{\partial \bar{R}_{i,1}^l(L_{i,1})}{\partial L_{i,1}}$ is given by totally differentiating the rewritten equation $L_{i,1} = \left[\frac{(1 - \tau_{i,1}^s) \bar{R}_{i,1}^l}{(L_1)^{\alpha-1 + \frac{1}{\rho}} \alpha \bar{A}} \right]^{-\rho}$

$$(1 - \tau_{i,1}^s) \bar{R}_{i,1}^l = (L_{i,1})^{-\frac{1}{\rho}} (L_1)^{\alpha-1 + \frac{1}{\rho}} \alpha \bar{A}$$

with respect to $L_{i,1}$ and taking into account the fact $\frac{\partial L_t}{\partial L_{i,t}} = \frac{1}{n} \left(\frac{L_t}{L_{i,t}} \right)^{\frac{1}{\rho}}$.

$$\frac{\partial \bar{R}_{i,1}^l}{\partial L_{i,1}} = - \left[\frac{L_1}{L_{i,1}} - \frac{1}{n} \left(\frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}} ((\alpha - 1) \rho + 1) \right] \frac{1}{\rho} L_1^{-1} \bar{R}_{i,1}^l$$

If the net worth constraint does not bind in $t = 1$ ($\lambda_{i,1} = 0$), then the first order condition with respect to $L_{i,1}$ becomes

$$\bar{R}_{i,1}^l \left(1 - \left[1 - \frac{1}{n} \left(\frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}-1} ((\alpha - 1) \rho + 1) \right] \frac{1}{\rho} \right) - (1 + \tau_{i,1}^{cc}) R_1^f = 0$$

If the net worth constraint is not binding, the first order condition is not a function of period zero variables. Since banks are symmetric, I consider the symmetric equilibrium which implies $(1 - \tau_{i,1}^s) \bar{R}_{i,1}^l = \tilde{R}_1^l$ and $L_{i,1} = L_1$. In a symmetric equilibrium

$$\tilde{R}_1^l = \gamma (1 - \tau_1^s) (1 + \tau_1^{cc}) R_1^f$$

where $\gamma = \frac{1}{(1 - \frac{1}{\rho}(1 - \frac{1}{n}) - (1 - \alpha)\frac{1}{n})}$ is the mark-up.

If the net worth constraint binds in $t = 1$ ($\lambda_{i,1} > 0$), the amount of loans in period one becomes

$$L_{i,1} = \frac{1}{\eta} N_{i,1}$$

where

$$\lambda_{i,1} = \frac{1}{\eta} \left[\bar{R}_{i,1}^l \left(1 - \left[1 - \frac{1}{n} \left(\frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}-1} ((\alpha - 1) \rho + 1) \right] \frac{1}{\rho} \right) - (1 + \tau_{i,1}^{cc}) R_1^f \right]$$

$$N_{i,1}(s_L) = A_L (L_0)^\alpha \frac{L_{i,0}}{\hat{L}_0} - (1 + \tau_{i,0}^{cc}) R_0^f [L_{i,0} - N_{i,0}] + T_{i,1}(s_L)$$

$$N_{i,1}(s_H) = \bar{R}_{i,0}L_{i,0} - (1 + \tau_{i,0}^{cc}) R_0^f [L_{i,0} - N_{i,0}] + T_{i,1}(s_H)$$

where $T_{i,1}(s_L) = \tau_{i,0}^{cc} R_1^f [L_{i,0} - N_{i,0}]$ and $T_{i,1}(s_H) = \tau_{i,0}^{cc} R_1^f [L_{i,0} - N_{i,0}] - \tau_{i,0}^s \bar{R}_{i,0}^l L_{i,0}$.

In a symmetric equilibrium, $\bar{R}_{i,1}^l = \frac{\bar{R}_1^l}{(1 - \tau_1^s)} = \frac{\alpha \bar{A}(L_1)^{\alpha-1}}{(1 - \tau_1^s)}$

$$L_1 = \frac{1}{\eta} N_1 \quad (\text{B.1.17})$$

where $\lambda_1 = \frac{1}{\eta} \left[\frac{\bar{R}_1^l}{(1 - \tau_1^s)} \frac{1}{\gamma} - (1 + \tau_1^{cc}) R_1^f \right]$.

Next, I solve the optimization problem of the banker in $t = 0$. Since, in equilibrium, there will be no default by banker i in $t = 1$ and considering only parametrization where the net worth constraint binds in the low state in $t = 1$ and does not bind in the high state in $t = 1$, banker i maximizes the expected dividend payment in the last period (notice that it is never optimal to pay dividends before $t = 2$).

$$\begin{aligned} & \max_{L_{i,0}} E_0 N_{i,2} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}] \\ = & \max_{L_{i,0}} \pi_H \left[\bar{R}_{i,1}^l (L_{i,0}; s_H) - (1 + \tau_{i,1}^{cc}(s_H)) R_1^f \right] L_{i,1}(L_{i,0}; s_H) \\ & + (1 - \pi_H) \left[\bar{R}_{i,1}^l (L_{i,0}; s_L) - (1 + \tau_{i,1}^{cc}(s_L)) R_1^f \right] L_{i,1}(L_{i,0}; s_L) \\ & + \pi_H (1 + \tau_{i,1}^{cc}(s_H)) R_1^f \left[\bar{R}_{i,0} - (1 + \tau_{i,0}^{cc}) R_0^f \right] L_{i,0} \\ & + (1 - \pi_H) (1 + \tau_{i,1}^{cc}(s_L)) R_1^f \left[A_L(L_0)^\alpha \frac{1}{\hat{L}_0} - (1 + \tau_{i,0}^{cc}) R_0^f \right] L_{i,0} \\ & + E_0 (1 + \tau_{i,1}^{cc}(s_1)) R_1^f \left[(1 + \tau_{i,0}^{cc}) R_0^f N_{i,0} + T_{i,1}(s_1) \right] + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}] \end{aligned}$$

Since in $t = 1$ the problem is static in the states of nature where the net worth constraint is not binding and I consider parametrization where it is not binding in the high state in $t = 1$, $L_{i,0}$ will not affect $L_{i,1}(s_H)$ and $R_{i,1}(s_H)$. With that in mind, the first order condition with respect to $L_{i,0}$ becomes

$$\begin{aligned} & (1 - \pi_H) \left(\frac{\partial \bar{R}_{i,1}^l(s_L)}{\partial L_{i,0}} L_{i,1}(s_L) + \left(\bar{R}_{i,1}^l(s_L) - (1 + \tau_{i,1}^{cc}(s_L)) R_1^f \right) \frac{\partial L_{i,1}(s_L)}{\partial L_{i,0}} \right) \\ & + \pi_H (1 + \tau_{i,1}^{cc}(s_H)) R_1^f \left(\bar{R}_{i,0} + \frac{\partial \bar{R}_{i,0}^l}{\partial L_{i,0}} L_{i,0} \right) - E_0 (1 + \tau_{i,1}^{cc}(s_1)) (1 + \tau_{i,0}^{cc}) R_1^f R_0^f \\ & + (1 - \pi_H) (1 + \tau_{i,1}^{cc}(s_L)) R_1^f \left[A_L(L_0)^\alpha \frac{1}{\hat{L}_0} + \left[\alpha A_L(L_0)^{\alpha-1} \frac{1}{\hat{L}_0} \frac{\partial L_0}{\partial L_{i,0}} - \frac{1}{n} A_L(L_0)^\alpha \frac{1}{\hat{L}_0^2} \right] L_{i,0} \right] \\ -\lambda_{i,0}\eta & = 0 \end{aligned}$$

where

$$\frac{\partial \bar{R}_{i,1}^l(s_L)}{\partial L_{i,0}} = \frac{\partial \bar{R}_{i,1}^l(s_L)}{\partial L_{i,1}} \frac{\partial L_{i,1}}{\partial L_{i,0}} = - \left[\frac{L_1}{L_{i,1}} - \frac{1}{n} \left(\frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}} ((\alpha - 1)\rho + 1) \right] \frac{1}{\rho} L_1^{-1} \bar{R}_{i,1}^l \frac{1}{\eta} \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}}$$

$$\frac{\partial L_{i,1}(s_L)}{\partial L_{i,0}} = \frac{1}{\eta} \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} \quad (\text{B.1.18})$$

$$\frac{\partial \bar{R}_{i,0}^l}{\partial L_{i,0}} = - \left[\frac{L_0}{L_{i,0}} - \frac{1}{n} \left(\frac{L_0}{L_{i,0}} \right)^{\frac{1}{\rho}} ((\alpha - 1)\rho + 1) \right] \frac{1}{\rho} L_0^{-1} \bar{R}_{i,0}^l$$

$$\frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = A_L L_0^\alpha \frac{1}{\hat{L}_0} + A_L \frac{1}{n} \left(\frac{L_0}{L_{i,0}} \right)^{\frac{1}{\rho}} \alpha (L_0)^{\alpha-1} \frac{L_{i,0}}{\hat{L}_0} - A_L L_0^\alpha \frac{L_{i,0}}{\hat{L}_0^2} \frac{1}{n} - (1 + \tau_{i,0}^{cc}) R_0^f$$

$$N_{i,1}(s_L) = A_L (L_0)^\alpha \frac{L_{i,0}}{\hat{L}_0} - (1 + \tau_{i,0}^{cc}) R_0^f [L_{i,0} - N_{i,0}]$$

$$N_{i,1}(s_H) = \bar{R}_{i,0}^l L_{i,0} - (1 + \tau_{i,0}^{cc}) R_0^f [L_{i,0} - N_{i,0}]$$

Assume that $\lambda_{i,0} = 0$. In a symmetric equilibrium $L_t = L_{i,t}$ and $(1 - \tau_t^s) \bar{R}_{i,t}^l = \tilde{R}_t^l$, $\bar{R}_{i,0}^l = \frac{\tilde{R}_0^l}{(1 - \tau_0^s)} = \frac{\alpha A_H (L_0)^{\alpha-1}}{(1 - \tau_0^s)}$, $\bar{R}_{i,1}^l(s_1) = \frac{\tilde{R}_1^l(s_1)}{(1 - \tau_1^s(s_1))} = \frac{\alpha \bar{A}(L_1(s_1))^{\alpha-1}}{(1 - \tau_1^s(s_1))}$ we get

$$MC(L_0) = -(1 - \pi_H) \left(\frac{1}{\gamma} \frac{\alpha \bar{A}(L_1(s_L))^{\alpha-1}}{(1 - \tau_1^s(s_L))} - (1 + \tau_1^{cc}(s_L)) R_1^f \right) \frac{1}{\eta} \left(\begin{array}{c} (1 - \frac{1}{n}(1 - \alpha)) A_L L_0^{\alpha-1} \\ - (1 + \tau_0^{cc}) R_0^f \end{array} \right) \quad (\text{B.1.19})$$

$$\begin{aligned} &= \pi_H (1 + \tau_1^{cc}(s_H)) R_1^f \frac{1}{\gamma} \frac{\alpha A_H (L_0)^{\alpha-1}}{(1 - \tau_0^s)} - E_0 (1 + \tau_1^{cc}(s_1)) (1 + \tau_0^{cc}) R_1^f R_0^f \\ &\quad + (1 - \pi_H) (1 + \tau_1^{cc}(s_L)) R_1^f \left(1 - (1 - \alpha) \frac{1}{n} \right) A_L (L_0)^{\alpha-1} = MB(L_0) \end{aligned} \quad (\text{B.1.20})$$

B.1.5 Proofs

Proposition 2.2.2: (i) If parametrization is such that there is a crisis in the low state in $t = 1$ and no crisis in the high state in $t = 1$ and in $t = 0$, $\lambda_1(s_L) > 0$, $\lambda_1(s_H) = 0$, $\lambda_0 = 0$, the equilibrium is unique and

exists. (ii) Countries with more competitive banking sector will borrow and invest more than countries with less competitive banking sector, $\frac{\partial L_0^*}{\partial \rho} > 0$, if

$$\underbrace{-(1 - \pi_H) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} \frac{\partial \lambda_{i,1}(s_L)}{\partial \rho}}_{=\frac{MC(\rho; L_0^*)}{\partial \rho} > 0} < \underbrace{R_1^f \pi_H \frac{\partial^2 N_{i,1}(s_H)}{\partial L_{i,0} \partial \rho}}_{=\frac{MB(\rho; L_0^*)}{\partial \rho} > 0} \quad (\text{Assumption 2.1})$$

and $\frac{\partial L_0^*}{\partial \rho} < 0$ if Assumption 2.1 is not satisfied.

Proof of Proposition 2.2.2: (i) In order to prove existence and uniqueness I will prove that $MB'(L_0) - MC'(L_0) < 0$. From equation 2.2.10

$$\begin{aligned} \frac{\partial MB}{\partial L_0} - \frac{\partial MC}{\partial L_0} &= R_1^f E_0 \frac{\partial N_1(s_1)}{\partial L_0 \partial L_0} + \\ (1 - \pi_H) \left[\lambda_1(s_L) \frac{\partial N_1(s_L)}{\partial L_0 \partial L_0} + \frac{\partial \lambda_1(s_L)}{\partial L_0} \frac{\partial N_1(s_L)}{\partial L_0} \right] &< 0 \end{aligned}$$

where from equations 2.2.13 and 2.2.12

$$\frac{\partial N_1(s_L)}{\partial L_0 \partial L_0} = (\alpha - 1) A_L L_0^{\alpha-2} \left(\frac{1}{n} [\alpha - 1] + 1 \right) < 0$$

$$\frac{\partial N_1(s_H)}{\partial L_0 \partial L_0} = (\alpha - 1) \alpha A_H L_0^{\alpha-2} \frac{1}{\gamma} < 0$$

$$\frac{\partial N_1(s_L)}{\partial L_0} = A_L L_0^{\alpha-1} \left(\frac{1}{n} [\alpha - 1] + 1 \right) - R_0^f$$

$$\frac{\partial \lambda_1(s_L)}{\partial L_0} = \frac{1}{\eta} (\alpha - 1) \alpha \bar{A}(L_1(s_L))^{\alpha-2} \frac{1}{\gamma} \frac{1}{\eta} \frac{\partial N_1(s_L)}{\partial L_0}$$

Notice that it will be the case that either $\frac{\partial N_1(s_L)}{\partial L_0} < 0$ and $\frac{\partial \lambda_1(s_L)}{\partial L_0} > 0$ or $\frac{\partial N_1(s_L)}{\partial L_0} > 0$ and $\frac{\partial \lambda_1(s_L)}{\partial L_0} < 0$, which implies that in either case $\frac{\partial MB}{\partial L_0} - \frac{\partial MC}{\partial L_0} < 0$. Combined with the fact that $\lim_{L_0 \rightarrow 0} [MB(L_0) - MC(L_0)] \rightarrow \infty$ and $\lim_{L_0 \rightarrow \infty} [MB(L_0) - MC(L_0)] \rightarrow -\infty$, this is sufficient to prove existence and uniqueness. Since $MB > 0$ if $\lambda_1(s_L) > 0$, it will have to be the case that if the equilibrium exists, also $MC > 0$ which will imply that, in equilibrium, $\frac{\partial N_1(s_L)}{\partial L_0} < 0$.

(ii) Totally differentiate $MB(L_0) - MC(L_0) = 0$ with respect to ρ

$$MB(L_0) - MC(L_0) = R_1^f E_0 \frac{\partial N_{i,1}(s_1)}{\partial L_{i,0}} + (1 - \pi_H) \lambda_1(s_L) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = 0$$

$$\frac{\partial L_0^*}{\partial \rho} = \frac{-R_1^f \pi_H \frac{\partial^2 N_{i,1}(s_H; L_0^*)}{\partial L_{i,0} \partial \rho} - (1 - \pi_H) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} \frac{\partial \lambda_{i,1}(s_L; L_0^*)}{\partial \rho}}{\frac{\partial (MB(L_0) - MC(L_0))}{\partial L_0}}$$

where $\frac{\partial (MB(L_0) - MC(L_0))}{\partial L_0} < 0$. $\frac{\partial L_0^*}{\partial \rho} > 0$ if $-(1 - \pi_H) \frac{\partial N_{i,1}(s_L)}{\partial L_0} \frac{\partial \lambda_{i,1}(s_L; L_0^*)}{\partial \rho} < R_1^f \pi_H \frac{\partial^2 N_{i,1}(s_H; L_0^*)}{\partial L_{i,0} \partial \rho}$ and

$$\begin{aligned} \frac{MB(\rho; L_0^*)}{\partial \rho} &= R_1^f \pi_H \frac{\partial^2 N_{i,1}(s_H; L_0^*)}{\partial L_{i,0} \partial \rho} = -R_1^f \pi_H L_0^{*\alpha-1} \alpha A_H \frac{1}{\gamma^2} \gamma'(\rho) > 0 \\ \frac{MC(\rho; L_0^*)}{\partial \rho} &= -(1 - \pi_H) \frac{\partial N_{i,1}(s_L)}{\partial L_0} \frac{\partial \lambda_{i,1}(s_L; L_0^*)}{\partial \rho} \end{aligned} \quad (\text{B.1.21})$$

$$= (1 - \pi_H) \frac{1}{\eta} \left[\left(1 - (1 - \alpha) \frac{1}{n} \right) A_L L_0^{*\alpha-1} - R_0^f \right] \alpha \bar{A} (L_1^*(s_L))^{\alpha-1} \frac{1}{\gamma^2} \gamma'(\rho) > 0 \quad (\text{B.1.22})$$

where $\gamma'(\rho) < 0$.

Proposition 2.3.3: *If $n = 1$ and the net worth constraint binds only in the low state in $t = 1$ for both the CP and the monopolistic bank, then $L_0^{CP} > L_0^*(n = 1)$ if*

$$R_1^f (1 - \alpha) \pi_H A_H \alpha L_0^{*\alpha-1} + (1 - \pi_H) \frac{1}{\eta} \left(\alpha A_L (L_0^*)^{\alpha-1} - R_0^f \right) [1 - \alpha] \bar{A} (L_1^*(s_L))^{\alpha-1} > 0 \quad (\text{Assumption 2.3})$$

where $L_1^* = \frac{1}{\eta} \left(A_L (L_0^*)^\alpha - R_1^f [L_0 - N_0] \right)$. If Assumption 2.3 is violated, then the monopolist overinvests relative to the Central Planner $L_0^{CP} < L_0^*(n = 1)$.

Proof of Proposition 2.3.3: The proof is based on a local perturbation around the decentralized equilibrium L_0^* which is without loss of generality given that $MB'(L_0) - MC'(L_0) < 0$ and $MB^{CP'}(L_0) - MC^{CP'}(L_0) < 0$. Re-writing the first order conditions of the CP and the banker from the decentralized equilibrium, after imposing $n = 1$

$$MB(L_0) - MC(L_0) = R_1^f \left(\pi_H \alpha^2 A_H L_0^{\alpha-1} + (1 - \pi_H) A_L L_0^{\alpha-1} \alpha - R_0^f \right) \quad (\text{B.1.23})$$

$$+ (1 - \pi_H) \frac{1}{\eta} \left[\alpha^2 \bar{A} (L_1(s_L))^{\alpha-1} - R_1^f \right] \left[A_L L_0^{\alpha-1} \alpha - R_0^f \right] \quad (\text{B.1.24})$$

$$MB^{CP}(L_0) - MC^{CP}(L_0) = R_1^f \left(E_0 A_1 \alpha (L_0)^{\alpha-1} - R_0^f \right) \quad (\text{B.1.25})$$

$$+ (1 - \pi_H) \left[\alpha \bar{A}(L_1(s_L))^{\alpha-1} - R_1^f \right] \frac{1}{\eta} \left(\alpha A_L(L_0)^{\alpha-1} - R_0^f \right) \quad (\text{B.1.26})$$

Since $MB'(L_0) - MC'(L_0) < 0$ and $MB^{CP'}(L_0) - MC^{CP'}(L_0) < 0$, then $L_0^{CP} > L_0^*$ ($n = 1$) if

$$\begin{aligned} & MB^{CP}(L_0^*) - MC^{CP}(L_0^*) - (MB(L_0^*) - MC(L_0^*)) \\ &= R_1^f (1 - \alpha) \pi_H A_H \alpha L_0^{*\alpha-1} + (1 - \pi_H) \frac{1}{\eta} \left(\alpha A_L(L_0^*)^{\alpha-1} - R_0^f \right) [1 - \alpha] \bar{A}(L_1^*(s_L))^{\alpha-1} > 0 \end{aligned}$$

which is true conditional on Assumption 2.3 being satisfied.

Proposition 2.3.4: *For any n and ρ and if the net worth constraint binds only in the low state in $t = 1$ for both the CP and banker in the decentralized equilibrium, then $L_0^{CP} > L_0^*$ if*

$$\begin{aligned} & \underbrace{R_1^f \pi_H A_H \alpha \left(1 - \frac{1}{\gamma} \right) L_0^{*\alpha-1} - R_1^f (1 - \pi_H) A_L (1 - \alpha) \left(1 - \frac{1}{n} \right) L_0^{*\alpha-1}}_{\text{underinvestment in } t=0 \geq 0} + \underbrace{\left(1 - \pi_H \right) \frac{1}{\eta} \left[\alpha \bar{A}(L_1^*(s_L))^{\alpha-1} - R_1^f \right] (1 - \alpha) \left(1 - \frac{1}{n} \right) A_L L_0^{*\alpha-1}}_{\text{"bankruptcy" pecuniary externality} \leq 0} \quad (\text{Assumption 2.4}) \\ & - \underbrace{(1 - \pi_H) \frac{1}{\eta} \left[R_0^f - \left(1 - (1 - \alpha) \frac{1}{n} \right) A_L L_0^{*\alpha-1} \right] \alpha \bar{A}(L_1^*(s_L))^{\alpha-1} \left(1 - \frac{1}{\gamma} \right)}_{\text{"net worth constraint" pecuniary externality} \leq 0} \\ & - \underbrace{(1 - \pi_H) \frac{1}{\eta} \left[R_0^f - \left(1 - (1 - \alpha) \frac{1}{n} \right) A_L L_0^{*\alpha-1} \right] \alpha \bar{A}(L_1^*(s_L))^{\alpha-1} \left(1 - \frac{1}{\gamma} \right)}_{\text{underinvestment in } t=1 \leq 0} > 0 \quad (\text{B.1.27}) \end{aligned}$$

where $L_1^* = \frac{1}{\eta} \left(A_L(L_0^*)^\alpha - R_1^f [L_0 - N_0] \right)$. If Assumption 2.4 is violated, then the banker overinvests relative to the Central Planner $L_0^{CP} < L_0^*$.

Proof of Proposition 2.3.4: The proof is based on a local perturbation around the decentralized equilibrium L_0^* which is without loss of generality given that $MB'(L_0) - MC'(L_0) < 0$ and $MB^{CP'}(L_0) - MC^{CP'}(L_0) < 0$. Re-writing the first order conditions of the banker from the decentralized equilibrium

$$\begin{aligned} MB(L_0) - MC(L_0) &= R_1^f \left(\pi_H (L_0)^{\alpha-1} \alpha A_H \frac{1}{\gamma} + (1 - \pi_H) A_L L_0^{\alpha-1} \left(\frac{1}{n} [\alpha - 1] + 1 \right) - R_0^f \right) \\ &+ (1 - \pi_H) \frac{1}{\eta} \left[\left((\alpha - 1) \frac{1}{n} + 1 \right) A_L L_0^{\alpha-1} - R_0^f \right] \left[\alpha \bar{A}(L_1(s_L))^{\alpha-1} \frac{1}{\gamma} - R_1^f \right] \end{aligned}$$

Since $MB'(L_0) - MC'(L_0) < 0$ and $MB^{CP'}(L_0) - MC^{CP'}(L_0) < 0$, then $L_0^{CP} > L_0^*$ if $MB^{CP}(L_0^*) - MC^{CP}(L_0^*) - (MB(L_0^*) - MC(L_0^*)) > 0$, which will be true if Assumption 2.4 is satisfied.

Proposition 2.4.1: Assuming the policy maker can commit and the net worth constraint binds in the crisis state, the CP's allocation can be decentralized using a lump sum transfer to entrepreneurs, T_t , subsidy on entrepreneurs' borrowing rates, $\tau_t^s \geq 0$, and a capital account control in the form of a tax on banker's borrowing rates from foreigners, $\tau_t^{cc} \geq 0$. One possible implementation of the constrained Central Planner's allocation is given by: $\tau_1^s = 1 - \frac{1}{\gamma}$, $\tau_1^{cc} = 0$. If $\tilde{\tau}_0^{cc}(\tau_0^s) > 0$, then $\tau_0^{cc} = \tilde{\tau}_0^{cc}(\tau_0^s)$ and $\tau_0^s = 0$. If $\tilde{\tau}_0^{cc}(\tau_0^s) < 0$, then $\tau_0^{cc} = 0$ and $\tau_0^s > 0$, where τ_0^s is pinned down by $\tilde{\tau}_0^{cc}(\tau_0^s) = 0$ and

$$\begin{aligned} \tilde{\tau}_0^{cc}(\tau_0^s) = & \underbrace{-\Phi R_1^f \pi_H \alpha A_H \left(1 - \frac{1}{\gamma(1 - \tau_0^s)}\right)}_{\text{period 0 underinvestment}} + \underbrace{\Phi R_1^f (1 - \pi_H) A_L (1 - \alpha) \left(1 - \frac{1}{n}\right)}_{\text{"bankruptcy" pecuniary externality} \geq 0} \\ & + \underbrace{\Phi (1 - \pi_H) \left[\alpha \bar{A} (L_1^{CP}(s_L))^{\alpha-1} - R_1^f\right] \frac{1}{\eta} A_L (1 - \alpha) \left(1 - \frac{1}{n}\right)}_{\text{"net worth constraint" pecuniary externality} \geq 0} \end{aligned} \quad (\text{B.1.28})$$

$\Phi = \frac{1}{(L_0^{CP})^{1-\alpha} R_0^f [R_1^f + (1-\pi_H) [\alpha \bar{A} (L_1^{CP}(s_L))^{\alpha-1} - R_1^f] \frac{1}{\eta}]} > 0$ and CP stands for the optimal allocation of the CP.

Proof of Proposition 2.4.1: In order to determine τ_1^s , compare the CP's first order condition with respect to L_1 , $\bar{R}_1^l = R_1^f$ to the first order condition of the banker with respect to L_1 from the Ramsey Problem, $\bar{R}_1^l = (1 - \tau_1^s)(1 + \tau_1^{cc})\gamma R_1^f$. Since $(1 - \tau_1^s)(1 + \tau_1^{cc})\gamma = 1$ and $\tau_1^{cc} \geq 0, \tau_1^s \geq 0, \gamma \geq 1$, then $\tau_1^{cc} = 0$ and $\tau_1^s = 1 - \frac{1}{\gamma}$. In practice, the CP's allocation can be decentralized in many difference way. Here, I derive the general formula for the optimal $\tilde{\tau}_0^{cc}(\tau_0^s)$ as a function of $\tau_0^s \in [0, 1)$. Combine the the first order condition of the banker with respect to L_0 from the Ramsey Problem, equation B.1.19, with the first order condition of the CP, equation B.1.11. Taking into account the fact that $\tau_1^{cc} = 0$ and $\tau_1^s = 1 - \frac{1}{\gamma}$, one can derive $\tilde{\tau}_0^{cc}(\tau_0^s)$ specified in the proposition, where $L_1^{CP}(s_L) = \frac{1}{\eta} N_1^{CP}(s_L) = \frac{1}{\eta} \left(A_L (L_0^{CP})^\alpha - R_0^f [L_0^{CP} - N_0]\right)$.

Appendix C

Supplement to Chapter 3

C.1 Appendix

C.1.1 Minimum MSFE Out-of-Sample Test Statistics

The Theil's U Test (TU)

The TU test statistic is a minimum MSFE test defined as the square root of the MSFE of the structural model over the square root of the MSFE of the random walk model. *Therefore, a $TU < 1$ implies that the structural model outperforms the random walk model.* TU is often preferred for its simplicity and intuitive interpretation and its statistical significance is tested via a bootstrap. The test we use is a one-sided test.

The Diebold – Mariano/West Test (DMW)

The DMW test statistic can be considered an alternative to the TU test. It measures the statistical significance of the difference between the MSFE of the random walk model and that of the structural model. *A significant and positive DMW test implies that the structural model outperforms the random walk.* On the basis of both theoretical and simulation evidence, ?, ?, Clark and McCracken (2001), Clark and McCracken (2005) and Clark and West (2006) show that, when comparing nested models, the asymptotic DMW test statistic is undersized, which means that it may not detect statistical significance (i.e., that the structural model outperforms the random walk model) even when it exists. While Clark and West (2006) attribute the poor size of the asymptotic DMW test statistic to small-sample bias, ?, Clark and McCracken (2001) and Clark and McCracken (2005) claim that the asymptotic DMW is undersized because the limiting distribution of the DMW under the null hypothesis is not standard normal when nested models are compared. To correct for this problem, a number of studies opt for the bootstrapped DMW test statistic which does not assume

any distributional form. This is the approach we take in this paper. Again, we calculate the DMW test as a one sided test.

The Clark – West Test (CW)

To compensate for the fact that the asymptotic DMW test statistic is undersized under the null hypothesis when comparing nested models and to avoid the use of a bootstrap, Clark and West (2006) and Clark and West (2007) propose a new asymptotic test for nested models, the CW, that builds on the asymptotic DMW test. The CW test statistic takes into account the fact that the two models compared are nested by assuming that, under the null hypothesis, the exchange rate follows a random walk.

When the forecast is calculated using rolling regressions, the limiting distribution of the CW under the null hypothesis is standard normal. However, when the estimation is performed recursively, the asymptotic distribution is approximated using Brownian motion. Based on simulation evidence, Clark and West (2007) suggest that, for recursive specifications, one can use a one-sided test and should reject the null hypothesis of equal forecasting power when the $CW \geq +1.282$ at 10 percent and $CW \geq +1.645$ at 5 percent (which are the critical values one would use assuming a normal distribution). Finally, we also calculate the bootstrapped CW test statistic to test whether the asymptotic CW test is properly sized.

The Clark - McCracken Test (ENC-NEW)

Another relatively new out-of-sample test statistic for nested models, the ENC-NEW, introduced by Clark and McCracken (2001) and Clark and McCracken (2005), also implicitly assumes that, under the null, the exchange rate follows a random walk.¹ The ENC-NEW and the CW differ only by a scaling factor. In other words, the two test statistics can differ slightly because of different power or size but they test the same null hypothesis. The shortcoming of the ENC-NEW is that its asymptotic distribution is a function of both the in-sample and out-of-sample portion of the data which makes evaluation of statistical significance quite cumbersome. Therefore, bootstrapping the ENC-NEW is an attractive alternative and this is the approach we take in this paper.

C.1.2 Proofs: The New Out-of-Sample Tests for Nested Models

¹Similarly to the CW, the ENC-NEW has been one of the most widely used out-of-sample test statistics in the exchange rate forecasting literature. Some of the studies that test out-of-sample forecastability using the ENC-NEW are ?, Rossi (2006), ?, ?.

In this section of the Appendix we provide a theoretical argument why the CW and ENC-NEW cannot be considered minimum MSFE tests in cases of severe "scale" forecast bias. We also argue that one should interpret these out-of-sample tests for nested models as prior tests of whether one can pool the random walk and the structural model forecast to produce a forecast with MSFE significantly smaller than the MSFE of the random walk.

From the Diebold – Mariano to the Clark – West Test

Before proceeding to the proofs, we present a derivation of the Clark and West test statistic, as presented in Clark and West (2006). In order to simplify the notation, assume that the forecast is one period ahead and that the forecast variable is the change in the exchange rate. Assume that $y_t = s_t - s_{t-1}$, where s_t is the natural log of the exchange rate for period t . Also let X_t be a matrix of explanatory variables. We are interested in comparing the forecasting power of the following theoretical models:

$$\begin{aligned} \text{Driftless Random Walk Model:} \quad & y_t = e_{1,t}, \text{ and} \\ \text{Structural Model:} \quad & y_t = X_{t-1}b + e_{2,t}, \end{aligned}$$

where $e_{1,t}$ and $e_{2,t}$ are the unobservable innovation terms.

The CW test assumes that, under the null hypothesis, the exchange rate is a random walk, and therefore, the population parameter $b = 0$, and the forecast innovation terms are equal, that is, $e_{1,t+1} = e_{2,t+1}$. The models can be estimated by OLS using either recursive or rolling regressions. The estimated forecasts for the random walk and the structural model are $\hat{y}_{1,t+1} = 0$ and $\hat{y}_{2,t+1} = X_t\hat{b}_t$ respectively. Denoting P as the number of forecasts, T as the sample length, and R as the sample reserved to calculate the first forecast, we can rewrite the sample difference between the MSFE of the two models (which is the main component of the DMW test statistic) as:

$$P^{-1} \sum_{t=R+1}^{t=T} \hat{e}_{1,t+1}^2 - P^{-1} \sum_{t=R+1}^{t=T} \hat{e}_{2,t+1}^2 = 2P^{-1} \sum_{t=R+1}^{t=T} (y_{t+1}X_t\hat{b}_t) - P^{-1} \sum_{t=R+1}^{t=T} (X_t\hat{b}_t)^2.$$

Clark and West (2006) argue that under the null hypothesis $e_{1,t+1} = e_{2,t+1} = y_{t+1}$; and since Clark and West (2006) assume that the independent variables are not correlated with the theoretical disturbance terms,

it follows that $E(y_{t+1}X_t\hat{b}_t) = 0$.² Therefore, they argue that we should expect $\sum_{t=R+1}^{t=T}(y_{t+1}X_t\hat{b}_t) \approx 0$ for both the rolling and recursive specifications. However, due to small-sample bias, $-P^{-1}\sum_{t=R+1}^{t=T}(X_t\hat{b}_t)^2 < 0$. As a result, the sample difference of the MSFEs of the random walk and the structural model is negatively biased in favor of the random walk.

The fact that the DMW test is negatively biased under the null hypothesis implies that it favors the random walk. Therefore, Clark and West (2006) propose an "adjusted" DMW statistic, or the so-called CW statistic, which tests whether

$$\hat{d} = 2P^{-1} \sum_{t=R+1}^{t=T} (y_{t+1}X_t\hat{b}_t) \quad (\text{C.1.1})$$

is significantly greater than zero. If it is, the structural model outperforms the random walk. More formally, we can define the CW as

$$CW = \frac{P^{0.5}\hat{d}}{\sqrt{\Omega^{\hat{d}}}}$$

where $\Omega^{\hat{d}}$ is the variance of \hat{d} . In comparison, one can write the ENC-NEW test statistics as

$$ENC - NEW = \frac{P \sum_{t=R+1}^{t=T} (y_{t+1}X_t\hat{b}_t)}{\sum_{t=R+1}^{t=T} (y_{t+1} - X_t\hat{b}_t)^2}$$

which makes it clear that the CW and ENC-NEW differ only by a scaling factor and we would expect that they behave similarly.

New Out-of-Sample Test Statistics (CW and ENC-NEW) : Not Minimum MSFE Tests

While the CW (and to a lesser degree the ENC-NEW) are often used interchangeably with the older minimum MSFE tests, we argue that their use as minimum MSFE tests is based on a misinterpretation, and that they should not be used as a substitute for the TU and the DMW tests. We present a theoretical proof that the CW is not a minimum MSFE test.³ The proof can be easily generalized for the ENC-NEW out-of-sample test statistic.

Our proof is based on the rolling window specification ($R/P \rightarrow 0$ and R is fixed) which generalizes well the main point made by Clark and West (2006), namely, the presence of small-sample bias. The proof for

² $e_{1,t+1} = e_{2,t+1}$ implies $E(y_{t+1}X_t\hat{b}_t) = E(e_{1,t+1}X_t\hat{b}_t) = E(e_{2,t+1}X_t\hat{b}_t)$. By assumption, $E(e_{2,t+1}X_t) = 0$. Then if one assumed that underlying variables are independent, $E(e_{2,t+1}X_t\hat{b}_t) = E(e_{2,t+1}X_t)E(\hat{b}_t) = 0$.

³Clark and West themselves suggest that researchers should interpret the CW as a minimum MSFE test statistic (2007, pp. 297). Clark and McCracken (2001, 2005) do not make such a claim regarding the ENC-NEW.

the recursive case is similar.⁴ While the proof assumes that the benchmark model is the driftless random walk, it can be generalized to any nested-model specification.

Assume that all the variables are defined as above. In the rolling regression specification, the null hypothesis incorporates the presence of small-sample bias and can be defined as $Ee_{1,t+1}^2 = Ee_{2,t+1}^2(R)$. The alternative can be defined as $Ee_{1,t+1}^2 > Ee_{2,t+1}^2(R)$. The respective MSFEs are

$$Ee_{1,t+1}^2 = E(y_{t+1})^2, \text{ and} \quad (\text{C.1.2})$$

$$Ee_{2,t+1}^2(R) = E(y_{t+1} - X_t \hat{b}_t)^2 = E(y_{t+1})^2 - 2E y_{t+1} X_t \hat{b}_t + E(X_t \hat{b}_t)^2, \quad (\text{C.1.3})$$

where $x(R)$ implies that the variable x is a function of a rolling window of fixed size, R . In the rolling regression case (the extreme version of small-sample bias with respect to the structural model parameter), R is fixed, and \hat{b}_t never converges to b , regardless of the sample size, T . The larger R is, the smaller the small-sample bias is. Also, when R is fixed, then under mild assumptions, we would expect $y_{t+1} X_t \hat{b}_t$ to be a well-behaved *iid* random variable. Then, it follows that

$$p \lim(P^{-1} 2 \sum_{t=R+1}^{t=T} (y_{t+1} X_t \hat{b}_t)) = 2E(y_{t+1} X_t \hat{b}_t),$$

where the probability limit is defined with respect to $P \rightarrow \infty$. Given the rolling regression set-up, we proceed to prove that CW is not a minimum MSFE test statistic. In other words, a statistically significant CW test does not imply a statistically significant minimum MSFE test.

Proposition C.1.1 $2E(y_{t+1} X_t \hat{b}_t) > 0 \not\Rightarrow Ee_{1,t+1}^2 - Ee_{2,t+1}^2(R) > 0$.

Proof of Proposition C.1.1. From equations (C.1.2) and (C.1.3), if $2E(y_{t+1} X_t \hat{b}_t) \leq E(X_t \hat{b}_t)^2$ then $Ee_{1,t+1}^2 - Ee_{2,t+1}^2(R) \leq 0$. However $2E(y_{t+1} X_t \hat{b}_t) \leq E(X_t \hat{b}_t)^2$ can hold even if $2E(y_{t+1} X_t \hat{b}_t) > 0$. As a result, $2E(y_{t+1} X_t \hat{b}_t) > 0$ does not imply $Ee_{1,t+1}^2 - Ee_{2,t+1}^2(R) > 0$. ■

⁴The recursive specification can be analyzed in a framework either with or without small-sample bias. If we assume that small sample bias is present, even when \hat{b}_t is estimated using recursive regressions, as Clark and West (2006) argue, we would still expect under certain assumptions that $2P^{-1} \sum_{t=R+1}^{t=T} (y_{t+1} X_t \hat{b}_t) \approx 2E(y_{t+1} X_t \hat{b}_t)$. As a result, in the presence of small-sample bias, the proof we present generalizes to the recursive case.

In the case when $R \rightarrow \infty, P \rightarrow \infty$ (i.e., no small-sample bias is present), the proof we present still holds. However, this case is irrelevant, given that according to Clark and West (2006)'s null hypothesis, if small-sample bias was not an issue, the adjustment of the DMW the authors propose would not be justified (under the null, the negative bias would disappear since $b = 0$ and, as a result, $E(X_t b)^2 = 0$).

The question emerges how often we would expect the CW(ENC-NEW) and DMW(TU) to produce different results due to the fact that the two test statistics test a different null hypotheses. In other words, how often we would observe $0 < 2E(y_{t+1}X_t\hat{b}_t) \leq E(X_t\hat{b}_t)^2$ in practice. The condition $0 < 2E(y_{t+1}X_t\hat{b}_t) \leq E(X_t\hat{b}_t)^2$ implies that if we regress the observed exchange rate change on the structural model forecast and no constant, the estimated coefficient should be less than or equal to $\frac{1}{2}$ and greater than 0. This is equivalent to having a significantly biased forecast (if the forecast is unbiased the estimated coefficient should be 1).⁵

Forecast bias is a significant problem in the literature on exchange rate forecasting. Marcellino (2000) emphasizes the importance of taking into account forecast bias when applying encompassing test statistics. ? and Clements and Hendry (2006) investigate the theoretical relationship between structural breaks and forecast bias and find that structural breaks, which are fairly common in forecasting, can lead to forecast bias.

What Do the New Out-of-Sample Test Statistics for Nested Models Test?

Here we prove that, *in theory*, a significant CW test implies that one can pool the random-walk and the structural-model forecasts and obtain a combined forecast whose MSFE is smaller than that of the random walk. Again, this proof also applies to the ENC-NEW test statistic.

Similarly to the proof of Proposition C.1.1, we prove the statement above in the context of the rolling specification, which implies that there is small-sample bias under the null hypothesis. However, a similar proof can be presented with respect to the recursive specification. The proof here is generalized to any nested model specification where Model 1 is nested in Model 2. Let $y_{c,t+1} = \lambda y_{2,t+1}(R) + (1-\lambda)y_{1,t+1}(R)$, $0 \leq \lambda \leq 1$ be the combined forecast where λ is the weight on the structural model forecast. Subscripts represent the respective model (1 or 2) and c stands for "combined". As before, the variable that we forecast is y_{t+1} . One can rewrite the CW test statistic as testing whether

$$\hat{d} = 2P^{-1} \sum_{t=R+1}^{t=T} \hat{e}_{1,t+1}(\hat{e}_{1,t+1} - \hat{e}_{2,t+1})$$

is significantly greater than zero. Within the more general framework of any nested model specification, we prove that a significant CW implies that there exists an optimal combination between the two forecasts which will produce a combined forecast that outperforms the simpler model (Model 1) in terms of MSFE.

⁵Note that the analysis refers only to "scale" bias since no constant is included in the forecast bias regression (for details see Marcellino (2000) and Holden and Peel (1989)).

Proposition C.1.2 $2Ee_{1,t+1}(R)(e_{1,t+1}(R) - e_{2,t+1}(R)) > 0 \Rightarrow \exists \lambda \text{ s. t. } Ee_{1,t+1}^2(R) - Ee_{c,t+1}^2(R) > 0$

Proof of Proposition C.1.2. The proof we present is similar to the proof provided by Harvey, Leybourne, and Newbold (1998). In a rolling regression framework and under mild assumptions

$$p \lim(2P^{-1} \sum_{t=R+1}^T \hat{e}_{1,t+1}(\hat{e}_{1,t+1} - \hat{e}_{2,t+1})) = 2Ee_{1,t+1}(R)(e_{1,t+1}(R) - e_{2,t+1}(R)), P \rightarrow \infty$$

We can minimize the MSFE of the combined forecast, by regressing y_{t+1} , the observed series, on $y_{1,t+1}(R)$ and $y_{2,t+1}(R)$ using *OLS* and constraining the coefficients to sum to one.

$$y_{t+1} = \lambda y_{2,t+1}(R) + (1 - \lambda)y_{1,t+1}(R) + e_{c,t+1}(R), 0 \leq \lambda \leq 1 \quad (\text{C.1.4})$$

If $\lambda > 0$, then combining the forecasts will produce a forecast s.t. $Ee_{1,t+1}^2(R) > Ee_{c,t+1}^2(R)$. Equation (C.1.4) can be rewritten as

$$e_{1,t+1}(R) = \lambda(e_{1,t+1}(R) - e_{2,t+1}(R)) + e_{c,t+1}(R), \quad (\text{C.1.5})$$

If we estimate equation (C.1.5) without a constant, then

$$\lambda = \frac{Ee_{1,t+1}(R)(e_{1,t+1}(R) - e_{2,t+1}(R))}{E(e_{1,t+1}(R) - e_{2,t+1}(R))^2}$$

Testing whether $2Ee_{1,t+1}(R)(e_{1,t+1}(R) - e_{2,t+1}(R)) = 0$ is testing the same hypothesis as testing whether $\lambda = 0$ using equation (C.1.4) or (C.1.5). Therefore, $2Ee_{1,t+1}(R)(e_{1,t+1}(R) - e_{2,t+1}(R)) > 0 \Rightarrow \exists \lambda \text{ s. t. } Ee_{1,t+1}^2(R) - Ee_{c,t+1}^2(R) > 0$ ⁶ ■

As a result, while the CW and the ENC-NEW out-of-sample test statistics cannot be considered minimum MSFE test statistics, they still provide meaningful information. They can be used as a prior test of whether a combined forecast exists that outperforms the driftless random walk forecast in terms of MSFE.

⁶One can also think of CW and ENC-NEW in the framework of encompassing test statistics. If one fails to reject the null that \hat{d} is equal to zero, then the random walk encompasses the structural model. If one rejects the null that \hat{d} is equal to zero, then the CW test statistic is statistically significant and the random walk fails to encompass the structural model. Note that a significant CW test statistic *does not* necessarily imply that the structural model encompasses the random walk. The distinction is important. If the structural model encompasses the random walk, which would occur if we fail to reject the null that $Ee_{2,t+1}(e_{2,t+1} - e_{1,t+1})$ equals 0, then the structural model will have a smaller MSFE than the random walk. As a result, encompassing will entail MSFE dominance (for proof see ? and Marcellino (2000)).

C.1.3 Bootstrap

Different Bootstrapping Procedures

First, we briefly discuss alternative approaches to bootstrapping. The standard bootstrap (also known as "case" bootstrap with replacement), introduced by ?, assumes that the re-sampled data are independent and identically distributed (iid). If the data are serially correlated or if heteroskedasticity is present (which is usually the case in time series data), a simple "case" bootstrap leads to inconsistent results. A better way to bootstrap time series data would be via a block bootstrap which implies that the data is re-sampled in blocks of a certain length rather than observation by observation, thereby preserving the properties of the data generating process (DGP). However, finding the optimal block size to preserve the DGP is not that straight forward. Indeed, Berkowitz and Kilian (2000) suggest that block bootstrapping is not the optimal way to bootstrap time series data given the state of development of the block bootstrap literature.

As a better alternative, Berkowitz and Kilian (2000) suggest a residual bootstrap procedure which is the bootstrap specification implemented by Mark (1995) and subsequently improved by Kilian (1999) and Mark and Sul (2001). The idea of residual bootstrapping is that in specifications such as Mark (1995) where the independent variable is defined as the deviation of the exchange rate from the fundamental, the cointegration (or the lack of cointegration) will be preserved when one uses the residual bootstrap. One way to implement it is to estimate an error correction specification, then re-sample the estimated residuals and recursively simulate the independent variable. (This type of bootstrap is commonly referred to as "semi-parametric" bootstrap. If one draws the residuals from a normal distribution, the bootstrap will be called "parametric".) While not always easy to implement, if properly specified, the bootstrap automatically corrects for small-sample bias and can be also used for forecast horizons greater than one as discussed in Kilian (1999).

Bootstrap Used in the Paper

The bootstrap procedure used to calculate the p-values of DMW, TU, CW and ENC-NEW for all specifications is similar to the bootstrap of Mark and Sul (2001) and ?. The main difference between our bootstrap and Mark and Sul (2001)'s bootstrap is that we use country specific OLS - regressions rather than seemingly unrelated regressions (SURs). We also perform a "semi-parametric" rather than "parametric" bootstrap. The data generating process (DGP) is a country – specific error correction process. The assumption of no unit root (or cointegration between the fundamental and the exchange rate) is imposed.

For each country, we estimate the following equations using an OLS regression (the “*i*” subscript is dropped for simplicity) :

$$\Delta s_t = \varepsilon_t^s$$

$$\Delta z_t = \mu + t + \gamma z_{t-1} + \sum_{k=1}^d \delta_k \Delta s_{t-k} + \sum_{k=1}^l \zeta_k \Delta z_{t-k} + \varepsilon_t^z$$

where z_t is the deviation of the exchange rate from the fundamental (or simply the fundamental) defined in (3.4.2), (3.4.3), (3.4.4) and (3.6.1). s_t is the nominal exchange rate. We also define $\Delta s_t = s_t - s_{t-1}$, $\Delta z_t = z_t - z_{t-1}$, μ is a constant and t is a trend.

Lags of Δs_t and Δz_t are included in the error correction equation to account for potential autocorrelation. The bootstrap procedure uses the Akaike's information criterion (AIC) method in order to choose between the appropriate number of lags of Δs_t and Δz_t (d and l can differ). The DGP can also differ across countries depending on whether AIC picks a specification with no constant, with constant or with a constant and a trend. The restriction that the sum of the coefficients of the lags of Δz_t equals one is imposed in order to avoid exploding simulated series. Then we re-sample the estimated matrix of residuals, $(\varepsilon^s, \varepsilon^z)$, either case by case (or more precisely row by row) or in blocks of 4 for quarterly data and 12 for monthly data. The results are relatively robust to the alternative methods of re-sampling of the residuals. Therefore, the results presented in the paper are based on case by case re-sampling of the residuals rather than block re-sampling.

Once the residuals are re-sampled, the exchange rate and the independent variable(s) are simulated recursively. The first 100 simulated observations are discarded in order to attenuate potential bias related to choosing the starting values of the recursion - the sample averages. With the new generated sample, the forecasting model is re-estimated and the test statistics are calculated. The p-values of the DMW, CW and ENC-NEW test statistics are measured as the portion of the distribution above the test statistics estimated using the observed data (since all these tests are one-sided tests), while the p-value of the TU statistic is the proportion of the bootstrapped TU distribution below the estimated TU value using the observed data. All the bootstrapped p -values are calculated on the basis of 1000 simulated distributions.